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Volume XXXI Number 7

Edited by William David Reeve

The Nature and Place of Objectives in Teaching Geometry

By E. R. Breslich
The University of Chicago

Mathematics in competition with other school subjects.—Up to the year 1910 the popularity of mathematics in the high schools of this country was on the increase. Not only was the number of pupils taking mathematics growing but, there was also a constant gain in the percentage of pupils in mathematics. In 1910 more than half of the pupils took algebra and about one-third took plane geometry. The increase took place in the face of a great deal of adverse criticism which often amounted to attacks on the subject.

The teachers of mathematics were not indifferent to these criticisms, and some reform movements were started to improve the teaching of mathematics, such as the correlation of mathematical subjects, the introduction of the laboratory method, the use of concrete materials in the development of the abstract principles of mathematics, and the inclusion of practical applications.

Since 1910 the percentage of pupils taking high-school mathematics has constantly dropped. In 1928[1] it had decreased to 35 per cent in algebra and to 20 per cent in plane geometry. This seems to indicate that mathematics is not meeting the competition with other high-school subjects as successfully as formerly.

Mathematics is being criticized.—Various reasons may be given for the decrease in popularity. Destructive criticisms and attacks upon the subject are probably responsible to a large degree. The criticisms have come from the following groups:

1. The parents of pupils who find mathematics more difficult than other subjects or have failed to make passing grades in it.

2. The people in the community who make little use of mathematics in their vocations, professions, and daily activities, and who try to cover up their deficiency by boasting that a knowledge of mathematics is something people can easily dispense with.

3. The teachers of other high-school subjects in which a knowledge of mathematics is needed and who protest loudly that pupils make mathematical mistakes or do not know how to use their mathematics.

4. The teachers of colleges and universities who complain that high-school graduates do not retain the mathematics taught in high-school courses.

[1] *Biennial Survey of Education 1926–1928*, pp. 1057–58. United States Office of Education, Bulletin, 1930, No. 16. Washington, D.C.: Government Printing Office, 1930.

5. People engaged in business and industry who find high-school graduates deficient in the simple mathematics needed in their special lines of work.

6. Administrators, psychologists, and general educators. Their criticisms are especially effective because they advise pupils as to the choice of subjects they are to take and because they have a great deal of influence upon the curriculum of the high school.

Most of the criticisms coming from the foregoing groups are destructive, vague, and of only slight help to the teacher. They either question the values of mathematics or claim that the values are not attained by the pupils.

Algebra has received a larger share of criticisms than geometry. The criticisms of teachers in other subjects, of teachers of the advanced mathematical courses, and of business men are nearly all directed against arithmetic and algebra. The reason is not that geometry is better taught, but that errors and deficiencies in arithmetic and algebra are readily detected, and that progress is blocked until they have been removed.

The need of objectives in teaching geometry.—Instruction in geometry is offered in all levels of school work: in the elementary school, the junior high school, the high school, and in the college. The term, as used in this paper, refers to the "demonstrative geometry" which is commonly taught in the second year of the high school. Those pupils who understand it often proclaim it to be the most fascinating and challenging of all courses. Many adults also express the opinion that of all high-school subjects they liked geometry best and that the teacher of geometry was the best teacher in the school. On the other hand, the subject is extremely difficult for a large percentage of pupils. Not a few teachers of mathematics are convinced that it is the most difficult course to teach.

Principals and supervisors who have the best opportunity to observe classes in geometry and who analyze teaching procedures, frequently express the opinion that it is the most poorly taught of all the mathematical subjects. Not that the teachers are lacking in knowledge of subject matter, but rather because they fail to appreciate the importance of clearly defined objectives. Purposeless teaching is vague and wasteful as to class time. A visit to such a class is disappointing because neither the visitor nor the pupils are able to acquire a clear notion as to what the teacher is expecting to accomplish.

In striking contrast to this is the work of the teacher who goes before the class with definite objectives in his mind. He does not permit discussion to sidetrack him, but proceeds as directly as possible toward the goals that he has set up. He is quick in sensing the opportunities that present themselves to aid the pupil in the attainment of the objectives.

Often the purpose of listing objectives of a course is to use them as weapons with which to defend the subject and to break down unfriendly attitudes of school officials, parents, and pupils. This is entirely justifiable. For in a rather crowded curriculum every subject is necessarily in competition with all others. However, defense of a subject should not be the major purpose. A far more important function of objectives is to improve teaching. This is not always recognized. Of course, nobody claims that a list of objectives will automatically change a poor teacher into a good teacher. It can only be said that, other things being equal, the better of two teachers will be the one who understands and is familiar with all the objectives he should strive for in the course.

Overemphasis upon a single objective, however important and legitimate it might be, is usually not beneficial to a course. Everybody knows what happens to geometry if the only objective of teaching is the ability of pupils to reproduce fluently the proofs of propositions that are given in the textbook. Day after day the class period is spent by the pupil either re-

citing proofs or listening to the recitations of others. There is little or no time for original work because the burden to be carried increases from day to day and some of the great values to be derived from the study of geometry will be completely lost to the pupil.

Objectives may conveniently be classified as general mathematical objectives, subject objectives, course objectives, and unit objectives. Besides striving for these objectives teachers are expected to make definite contributions to the development of such habits as accuracy, neatness, use of correct language, precise expression, and reading with comprehension. Examples of general mathematical objectives are functional thinking, and appreciation of the values of mathematics in the history of the race and in everyday life. Development of spatial imagination and understanding of the logical structure of a science are subject objectives and deserve emphasis in both plane and solid geometry. Proficiency in the use of the drawing instruments and of the geometric theorems in practical applications are course objectives. Each chapter or unit has its own specific objectives.

It is not intended to present in this paper an exhaustive discussion of objectives. Indeed, some writers have questioned the value of listing objectives because they fear that such a list might only distract the teacher. Classroom observation seems to prove that this fear is not justified. Indeed, familiarity, not mere acquaintance, with a comprehensive list of objectives is essential to effective teaching. Such a demand made on the teacher is not nearly as great as the teacher's demand that the pupils be thoroughly familiar with a list of definitions, postulates, and propositions running into hundreds.

An analysis of textbooks on plane geometry and of articles found in the mathematical journals and yearbooks discloses an extensive list of objectives. A few on which there seems to be general agreement have been selected for the discussion which follows.

Acquisition of geometric knowledge.—It may seem trite to say that a major objective of the teaching of geometry is "acquisition of geometric knowledge." However, there is reason to believe that this objective does not receive in courses in plane geometry the attention which it deserves. To study geometry means first of all to study the properties of the space in which we live; to learn the meanings of the basic concepts; to know the theorems; and to understand the relationships which they express. It is not at all a rare experience to find that persons who have studied geometry in the high school are ignorant of some or most of these matters. A recent experience will illustrate what is meant.

A rather successful young lawyer came to a teacher of mathematics for assistance. He was working on an important case related to the taxes on a valuable piece of property. "Usually," he explained, "taxes on property are determined from the frontage. In this particular case the tax has been computed from the number of square feet in the lot. Am I absolutely right when I say that the number of feet on one side of the lot multiplied by the number of feet on the other side gives the number of square feet in the lot? Everything I do depends on the correctness of this rule." He was, of course, assured that there was no question about it, and an explanation was given similar to the one usually presented in geometry. He went away satisfied.

Experiences of this type make one wonder how it is possible that a man of more than average intelligence, who had a high-school and college education, had passed the Illinois state law examinations, and was successful in his profession could have absorbed so small an amount of geometry. He could recall a rule and used it to compute the area. But when he looked at the result he said to himself, "It cannot be true. Why, it is impossible." If he really understood the rule he would

not have lost faith in it just because the result was a larger number than he expected.

Knowledge of geometric facts is usually to be acquired in geometry by deductive proof. It could be obtained by procedures that are entirely intuitive and experimental. Many pupils have more faith in these methods than in deductive proofs. They will throw aside a perfectly good proof any time if the facts do not seem to conform with experience. It is good teaching in high-school geometry to prove experimentally those theorems with which pupils in general are known to have difficulty. The deductive proof should follow when there is evidence that the meaning of the theorem is understood.

Ability to use the facts and principles taught in geometry.—There is probably no better test of understanding and mastery than the ability to use acquired knowledge in new situations. The method of teaching facts through use prepares the pupil for this test. The earlier textbooks did not recognize this principle, and presented the definitions and axioms at the beginning of the course, often long before the pupil had any use for them. The newer books postpone the introduction of new terms to the time and place where they are needed and can be put to immediate use. Likewise, theorems are not presented until they are to be used in proofs of other theorems and exercises. Constructions also are taught just before they are to be used in new constructions, and are thus kept under review. The need for a large amount of drill is thereby eliminated.

Of great value are the applications of geometry, which relate the subject to other school subjects and to life situations. Euclid had no use for the practical applications of geometry. He left them out. In a modern course they are rated highly. They create favorable attitudes among pupils, especially the practically minded. They give the subject a flavor of reality. They strengthen the position of geometry on the curriculum.

There is danger, however, that the values to be derived from applications are lost if the situations in which they occur do not fall within the experiences of the pupils. Such applications use up time and effort which the teacher might better spend on the geometric principles which they are to clarify. The pupils fear them. Confidence in their geometric ability is being undermined. If applications are to aid understanding, they should occur in familiar situations which do not require too much explanation. They are not to be brought in at the expense of time allotted to geometry.

Applications taken from other school subjects are especially good. Artistic designs, laws from the field of science, and shop problems appeal to most pupils. Very interesting applications are found in household problems, the trades, engineering, and surveying. They should be distributed over the course so as to include some in each chapter. However, the teacher should be on guard against overdoing. The pupils are usually not greatly impressed by practical applications. When an adult is asked to state reasons why he enjoyed the study of geometry he usually fails to mention the practical applications among his reasons.

Proficiency in using the straightedge and compasses and in making freehand drawings.—From the time of Plato geometric constructions have been restricted to those that make use only of compasses and unmarked straightedge. The so-called fundamental constructions are the bases of all later constructions, and are therefore taught first. Whether a "construction" is made on paper or on the blackboard, it should always be made with care and with the use of the instruments. Geometric constructions become purposeful when they are applied to exercises taken from other school subjects such as designing in art and in the shops.

Most of the drawings that pupils are required to make in geometry are not constructions but merely illustrations. They should be neatly and carefully drawn so as to represent with a fair degree of accuracy the conditions stated in exercises and problems. Hence pupils should be well trained in making good "freehand" drawings with pencil or crayon.

In performing experiments, the use of instruments other than straightedge and compasses should be allowed. The marks on the ruler, the protractor, squared paper, and 60° and 45° triangles deserve a place in that type of work. They are also valuable as checking instruments to test the accuracy of geometry constructions.

Spatial imagination.—The diagrams of plane geometry are exhibited almost entirely on the pages of the textbooks or notebooks and on the blackboard. They are rarely shown in the surroundings in which they usually appear in the pupils' experiences of everyday life. Hence pupils receive but slight training in spatial imagination. As much as possible the facts of plane geometry and the relationships between lines, angles, and arcs should be observed in three-dimensional space. Parallel and perpendicular lines, angles, and planes may be seen in the classroom. Gothic windows are found on church buildings; networks of triangles on towers and bridges; and circles on decorative designs and machinery. One simple problem in surveying will teach more geometry than a whole set of textbook problems. If the pupil is to recognize the connections between the figures he studies in the textbook and those that appear in three-dimensional space, they must be pointed out to him in teaching. References to figures in space of three dimensions should be the rule rather than exception. The activities of estimating and measuring lines, angles, and surfaces train pupils in space imagination. There is no scarcity of material. The teacher who keeps this objective in mind will have no difficulty in finding illustrations of plane geometry figures in three-dimensional space.

Appreciation of geometry as an example of a science.—Deductive geometry is probably the best example with which to show how knowledge may be systematized and how laws and principles may be exhibited in an ordered scheme consistent with each other and interrelated. Brifly, geometry illustrates simply and effectively the meaning of a science. Like other sciences which the pupil studies—e.g., biology, physics, and chemistry—geometry starts with a list of definitions and postulates as basic facts. Upon them the pupil builds new facts as he encounters them in the progress of study. He appreciates the role of the definitions and postulates. He builds his geometry very much as the workman builds a house, by first gathering the stones for the foundation. After the foundation is laid he adds one story at a time until the dwelling is complete.

The pupil who fails to conceive geometry as a science looks upon it merely as a collection of propositions to be proved, memorized, and recited to the teacher. He forms a narrow and false conception of the subject. Under the right kind of teaching, however, he sees a purpose in everything he does. Geometry becomes a structure in which he demonstrates the propositions to his own satisfaction until the finished building stands before him. Disregard of this objective in teaching is fatal to the course.

Power to think and reason logically.—Much has been said and written about the value of geometry as a subject which develops power to think and to reason logically. Perfect models of thinking are presented daily in a simple manner with simple material exhibiting the processes of correct thinking. Furthermore, the pupil is made acquainted with the common errors in reasoning such as generalizing from insufficient data and using incorrectly concepts that have not been clearly defined.

Demonstrative geometry differs from

the intuitive geometry of the junior high school and elementary schooi mainly in the method of reasoning. Emphasis has been shifted from inductive to deductive thinking. Synthesis, analysis, and indirect reasoning are being stressed throughout the course. However, induction should also have a place in demonstrative geometry. It should not be entirely dropped, because it is the method of experimentation and generalization. By applying both deductive and inductive methods to geometric facts the pupil will be led to an appreciation and understanding of what is meant by logical thinking.

This achievement does not come about automatically. It is not sufficient merely to "expose" the pupil to perfect models of logical thinking. He must be taught the processes of reasoning as carefully and systematically as he is taught the geometric theorems and constructions. He cannot be hurried. Assimilation requires time. Training in the processes of thinking begins the first day and continues each day thereafter. It continues even in future mathematics courses. Indeed, in most of them the ability to think and reason abstractly is presupposed.

Some writers feel that the emphasis on the teaching of logic in geometry is being overdone; that the course is suffering from too much logic; and that satisfactory results could be attained in less than a year with a greatly reduced list of propositions. They assert that the pupils' interest and enthusiasm are checked because they cannot see any reason why they should go through a long-drawn-out sequence of propositions and tedious proofs leading to facts which they have known and used for years in mathematics as well as in other school subjects. To solve the problem it has been suggested that theorems of this type be established experimentally and added to the list of postulates, in the belief that a greatly reduced list of propositions will be sufficient to give the desired training in deductive reasoning.

Ability to reason in life situations.—Attention has been called so far to the importance and need of a definite set of objectives in the teaching of geometry and the meanings of five major objectives have been discussed in detail: Acquisition of geometric knowledge; ability to use the facts and principles of geometry; proficiency in the use of the drawing instruments; appreciation of geometry as a science; and power to reason logically. Objectives serve mainly two purposes: They improve teaching, and they aid the teacher in defending the subject. To a certain extent the first is included in the second. For, what better way is there to defend the subject than to improve the teaching?

No defense is needed for those whose work in trade or profession requires a knowledge of geometry. They would have studied it no matter how it is taught, although they may feel that there is room for improvement in teaching. The problem of the geometry teacher is to cultivate the friendly attitude of the large group of those who do not, or say that they do not, use geometry. Indeed, there are many in that group who are enthusiastic about the subject. They enjoyed it as a high-school subject. If they are asked why they liked geometry they usually give replies like the following: "It was a challenge to my ability;" or "The teacher made me think;" or "It gave me a thrill each time I succeeded with an exercise."

The social and practical values of geometry are not to be overlooked, but they appeal only to a relatively small group. The logical values are of importance to everybody. Many adults firmly believe that in the training in reasoning and attacking problems in geometry they received something that was of definite value and help to them later in their occupations and professions.

Furthermore, it will be conceded that clear thinking is a requirement to effective citizenship. To develop men and women who know how to think accurately is of great importance from the social

point of view. The processes of thinking are expressed in no clearer way than when they are applied to the simple materials of geometry.

There are good reasons then for stressing the logical values of geometry in teaching geometry as well as in defending the subject, especially in view of the fact that the most recent investigations we have indicate that with the right kind of teaching such training in thinking can be made to transfer to situations outside of the field of geometry.

However, ability to reason in life situations is a nongeometric objective and should therefore *not receive too much emphasis* in a course in geometry. It is entirely legitimate, in fact, it is most important to lead the pupil to see that in many ways the reasoning applied to geometry is the same as the reasoning required in life situations, and that it applies to certain social problems that arise in everyday life as well as to geometry. Practice of this type should improve his reasoning when he deals with geometric situations. For example, analyzing verbal nongeometric statements to determine the hypotheses and conclusions will clarify the meaning of these terms in the statements of theorems. Changing assertions such as "Only citizens are allowed to vote" to the "if-then" form will aid the pupil in changing geometric statements like "The base angles of an isosceles triangle are equal" to the "if-then" form. Practice in stating converses of verbal statements; in using the indirect method of proof; in making generalizations; in attacking problems; in organizing ideas logically; and in making fine distinctions is of social value and supplements similar practice with geometric facts and statements. Selections of the right kind of problems taken from life will help the pupil understand the processes of thinking employed in geometry just as much as the processes taught in geometry will be of help to him in solving certain problems of everyday life.

The teachers of geometry, however, must not claim too much for their subject. For, the reasoning in life situations is often unlike the type of reasoning required in geometry; and time should not be spent on types of reasoning which are of no special help in the study of geometry. In life situations the best solutions of problems are not always those that are derived by strict deductive reasoning. Decisions are influenced and often dictated by social customs, personal interests, and other extraneous factors. To prepare pupils to solve these problems is to train them in ways of thinking and reasoning not permitted in the study of geometry.

To illustrate, a certain man, like most of us, is interested each morning in the weather conditions. On a cold morning he finds that there is snow on the ground and ice on the windows. Logically he reaches the following conclusions: "Since there is snow on the ground, I must wear rubbers." "There is ice on the windows. Therefore it is very cold. Hence, I must wear a warm cap and heavy gloves." Then he leaves the house without rubbers, wearing a felt hat and kid gloves. He discards entirely his logical conclusions. They are useless for the simple reason that he wishes to be dressed the way his friends are dressed. Thus he solves the problem by doing the illogical, not the logical, thing.

I have a very good friend. He is well educated. He has no bad habits. He is a church member. As far as I can tell he has only one serious fault. He is too logical in his thinking. He makes his friends uncomfortable by constantly requesting them to define the terms they use in ordinary conversation. Hence they avoid him. Nobody likes to serve with him on the same committee because he has acquired a reputation of being too argumentative. In reality he is merely logical. His logic is at its best when he proves to his superiors that they are all wrong. Naturally he is not getting on as well as his friends who are less logical and whose minds are more

flexible, although he works much harder than any of them.

It is not to be inferred from these two illustrations that the training received in geometry is not valuable in solving the problems of everyday life. However, ability to solve social problems depends usually as much or more on understanding of social conditions than on ability to think logically. There is danger of carrying logical discussions and problems beyond the point where they aid either in solving social problems or in the study of geometry. We cannot save geometry by replacing it by a course in logic. Courses in logic may be badly needed in our schools, but they are not geometry.

Improving the organizations of geometric materials.—Attention was called at the beginning to the decrease in the popularity of geometry. In a way it is hard to understand this because people in general are interested in mathematics. They fully appreciate the practical values of the subject, and they enjoy almost any mathematical discussion which they can understand. If they disliked mathematics in the high school it probably was because they could not understand it.

It has been suggested that the first effort to arouse greater interest in mathematics should be to improve the teaching. It is no less important to solve the problem of improving the organization of the geometric materials for teaching purposes. In attempting the solution it is necessary to recognize that the pupils may be classified in two groups: A large group which has been increasing in the last decades and in which are found those who dislike deductive geometry because they cannot understand it; a second group, decreasing from year to year, which consists of pupils who study geometry because they enjoy it, and in many cases also because they plan to prepare for future work, in which a knowledge of geometry is essential, as in engineering.

Deductive geometry as it is now organized serves the second group in an admirable way. Those of the first group who dislike it may take it or leave it. Perhaps the time has come to build a second course in geometry which will appeal to the interests and which will serve the needs of the first group as well as Euclid's geometry has served the second for centuries. It should not be difficult to plan a very useful and attractive course.

Such a course would continue the good work in geometry that has been started in junior high-school mathematics. It should be practical above anything else, and those propositions that have the best applications should be the ones to be emphasized. The method of study should be inductive and experimental, although some deductive reasoning would have a place. Geometric constructions, drawing, and designing should receive much attention. A major objective would be training in space imagination and spatial relationships. While learning the principles of plane geometry the pupil would make frequent excursions into three-dimensional space. Most of the pupils taking such a course would probably be of the noncollegiate type, but many others would take it even though the colleges would not recognize the course for credit.

If schools could be induced to offer such a course in practical geometry, it would be possible to improve greatly the course in deductive geometry. Since most of the pupils who would take it would do so for a purpose, a good attitude would be assured. Pupils of low ability could be kept out, since they would be advised to take the practical geometry.

The way would thus be paved to carry out all the recommendations for improvement of deductive geometry which have been made in the last decades. First, the number of basic propositions for which all pupils are to be held responsible should be reduced to make room for new material that may be introduced.

Proofs and solutions of exercises should be simplified by using consistently algebraic notation, equations, and formulas.

Likewise trigonometry should be more

than just a chapter sandwiched in between two unrelated chapters. It should be used to simplify or replace certain geometric and algebraic proofs, to solve exercises, and to express relationships.

Axial and central symmetry should do more than describe symmetric figures. It should be used as a method of proof.

Excursions from plane into solid geometry should be frequent.

These recommendations have been made before, but the teachers have not and probably could not take them seriously because the traditional course has been overcrowded with materials and because the ability of many pupils in the class has been too low to permit rapid progress. With a better student body it should not be difficult to introduce some or all of these recommendations.

In closing, it will be in order to add to the foregoing recommendations one other: To introduce graphs and graphical methods. Many theorems of plane geometry can be proved in a simple manner by analytical methods. Graphs have been studied in first-year algebra; why not continue to use them in geometry? Next to trigonometry, analytic geometry is probably the most attractive course of elementary college mathematics. The use of the simple concepts of both would not only aid the pupil in the study of geometry, but would impress upon him the fact that the most interesting mathematics courses are still ahead of him.

Repeating Decimals

By P. H. Nygaard
North Central High School, Spokane, Wash.

When a fraction is converted into a decimal by long division, it often happens that the quotient contains a continuously repeating series of figures. Thus, $\frac{29}{110}=.2636363\cdots$, which has two figures in its repeating group. It should be noted that many repeating decimals do not start repeating immediately,—that is, the first figures after the decimal point may not be a part of the recurring series. On the other hand, a fraction may give an exact decimal value. For instance, $\frac{33}{80}=.4125$. These two possibilities are not exceptions; it is a well established mathematical law that all division problems in which the dividend and the divisor are integers will eventually give, as the quotient, either an exact decimal or a repeating decimal. The converse law is also true,—namely, for every exact decimal or repeating decimal it is possible to find a fraction, with integral terms, to which the decimal is equal.

These laws and the whole topic of repeating decimals receive little, if any, mention in high school and college mathematics courses. The topic belongs to the branch of mathematics known as the theory of numbers, which is generally conceded to contain more unproved theorems, to provide more treacherous footing, and to afford more promise for the future investigator than any other branch of the subject. The discussion here given is intended to be an elementary survey suitable for high school students and teachers. Since an attempt will be made to formulate the explanatory statements with rigorous exactness, the reader is warned to pay careful attention to all exceptions and provisions in order to avoid jumping at incorrect conclusions.

How to Tell Whether a Fraction Will Repeat

How can it be determined in advance whether m/n, in which m and n are integers relatively prime to each other, will give a repeating decimal or an exact decimal? The rule is very simple: If n is exactly divisible by no prime numbers except 2 or 5, the fraction will have an exact decimal value; if n has a prime factor other than 2 or 5, m/n will give a repeating decimal. According to this rule, $\frac{39}{160}$ will yield an exact decimal value, because $160=2^5\times5$; but $\frac{37}{150}$ will generate a repeating decimal, because 150 contains the prime factor 3. This rule obviously follows from the fact that 10, which is the base of our number system, contains no other prime factors than 2 and 5.

It may be natural at this point to ask how many decimal places will be necessary in order to complete an exact or a repeating decimal. When the rule in the previous paragraph indicates an exact value for an irreducible fraction, m/n, the number of decimal places in the result will be the highest power of either 2 or 5 that is contained in n. Thus, $\frac{33}{80}$ will have 4 decimal places, because $80=2^4\times5$; $\frac{381}{500}$ will have 3 decimal places, because $500=2^2\times5^3$. No simple rule is possible in case the fraction gives a repeating decimal, although the number of decimal places needed to come to the end of the first of the repeating groups will never exceed $(n-1)$. For the fraction $\frac{30}{7}$ it follows that 6 decimal places will suffice. Actual division gives $\frac{30}{7}=4.285714285714\cdots$, so here the full maximum is required.

Peculiarities of Repeating Decimals

The fraction $\frac{1}{7}=.142857142857\cdots$. This has a repeating group or, as it is usually called, a period of 6 figures,—namely, 142857. Some rather surprising things can be done with this number. Multiplying 142857 by 2 gives 285714; multiplying

142857 by 3 gives 428571. Each product contains the original figures 142857, but that is not all. The figures are also arranged in the same order as before, the only difference being that they start at another place in the sequence. It is understood that 7, the last figure, is followed by 1, the first figure. Similar results are obtained when the multiplier is 4, 5, or 6, but not for larger multipliers. This cyclic property shows up in another way. Changing $\frac{18}{7}$ to a repeating decimal gives $2.571428571428\cdots$. The period is now 571428, which is the same as for $\frac{1}{7}$ except that it starts with 5 instead of 1.

In general it may be stated that if $1/n$ has a repeating period, p, of $(n-1)$ figures, then the result obtained by multiplying p by any integer, greater than 1 and less than n, will contain the same figures and in the same order as in p, but will start with a different figure than did p; also, under the same condition as before, the repeating decimal obtained by dividing any integer, not a multiple of n, by n will have the same period as did $1/n$, but may start with a different figure in the period. There are many values of n in addition to 7 for which these cyclic properties are true. Among these values are $n=17$, 19, 23, 29, 47, 59, 61, and 97. For $\frac{1}{17}$ the period is 0588235294117647. This number may therefore, be multiplied by any integer from 2 to 16, inclusive, and the product in each case will be the original period starting with a different figure. For instance, multiplying it by 14 gives 8235294117647058. Also it will be found that $\frac{50}{17}=2.9411764705882352\cdots$.

Of course, not all prime values of n have a period of $(n-1)$ figures in the repeating decimal obtained from $1/n$. Thus, $\frac{1}{13}=.076923076923\cdots$, whose period has only 6 figures. For such repeating decimals the cyclic properties break down. It will be found that $076923\times2=153846$, which is entirely different from the original period. The break down is not complete, however because $076923\times3=230769$. For repeating decimals whose period has less than $(n-1)$ figures, some multipliers give the same circulating period and some give a different one; some values of m when divided by n yield the original cycle and others do not. No simple rules are available to explain this erratic behavior.

Another curious property of repeating decimals is well illustrated by 142857, the period for $\frac{1}{7}$. If any figure in the first half of the period be added to the corresponding figure in the second half, the sum is always 9. Thus, $1+8=9$, $4+5=9$, and $2+7=9$; or, in another form, $142+857=999$. This complementary relationship between the two halves of a period having an even number of figures is true whenever the generating fraction, m/n is irreducible and n is prime. It may, however, be true for certain other generating fractions. It is apparent that the following comply with the above rule: $\frac{6}{11}$ gives the period 54; $\frac{1}{13}$ gives the period 076923; $\frac{4}{19}$ gives the period 210526315789473684. The following do not comply with the rule, because n is not prime, but do, nevertheless, reveal the complementary relationship: $\frac{1}{26}$ gives the period 384615; $\frac{3}{14}$ gives the period 142857. To show that not all even numbered periods are complementary, take $\frac{1}{21}$, whose period is 047619.

There is a simple relationship involving the last figure of the period of a repeating decimal. This last figure, when the generating fraction is $1/n$, is always 1 or 9, if the unit's figure of n is 9 or 1, respectively. To illustrate: $\frac{1}{81}$ has the period 012345679; $\frac{1}{39}$ has the period 025641. If the unit's figure of n is not 1 or 9, the last figure of the period is the same as this unit's figure, provided however that n is prime. To illustrate: $\frac{1}{37}$ has the period 027; $\frac{1}{43}$ has the period 023255813953488372093.

Length of the Period

As has been already mentioned, the repeating decimal obtained from $1/n$ cannot have a period of more than $(n-1)$ figures. If n is prime and if x is the number of figures in the period of $1/n$, it can be

proved that x must either equal $(n-1)$ or be exactly divisible into $(n-1)$. Hence the number of figures in the period generated by $\frac{1}{13}$ must be 12, 6, 4, 3, 2, or 1; the number of figures in the period for $\frac{1}{23}$ must be 22, 11, 2, or 1. The first actually has 6 figures in its period and the second has 22.

Just which is the right value of x in a particular problem is a matter that has been given a great deal of thorough investigation without arriving at any rules that may be readily applied. The general theory is as follows: If n is prime and x is the number of figures in the repeating period for an irreducible fraction, m/n, then x is the least integral value that makes 10^x-1 exactly divisible by n. Suppose $m/n=\frac{24}{37}$. If $x=1$, $10^x-1=9$, which is not divisible by 37; if $x=2$, $10^x-1=99$, which is not divisible by 37; if $x=3$, $10^x-1=999$, which equals 37×27. Hence $x=3$, meaning that there will be 3 figures in the period for $\frac{24}{37}$. This was fairly easy, but if n had been, say, 29, the process would be exceedingly tedious. The smallest value of x that would make 10^x-1 divisible by 29 would be $x=28$. Even if this method were used only up to $x=14$, it would be necessary to divide $10^{14}-1$ by 29—that is, $99,999,999,999,999\div29$. Since this gives a remainder, the rule stated in the previous paragraph would enable us to conclude immediately that $x=28$. On the basis of this and related theories, lengthy tables have been computed from which x can be determined for prime values of n up to at least 10,000.

If n is composite, the number of figures, x, in the period for $1/n$ can be found from the prime factors of n. Suppose $n=2^c\times5^k\times n_1\times n_2\times n_3\cdots$, in which c and k can be any integers each equal to or greater than 0, and $n_1, n_2, n_3, \cdots$ are prime numbers all different from each other and none of which is 2 or 5. Also let the repeating decimal for $1/n_1$ have x_1 figures in its period, that for $1/n_2$ have x_2 figures in its period, etc. Then x is the lowest common multiple of $x_1, x_2, x_3, \cdots$.

To illustrate: $\dfrac{1}{14140}=\dfrac{1}{2^2\times5\times7\times101}\cdot$ The period for $\frac{1}{7}$ has 6 figures and that for $\frac{1}{101}$ has 4 figures. The lowest common multiple of 6 and 4 is 12. Hence the period for $\frac{1}{14140}$ will have 12 figures.

The preceding method fails if any of the numbers $n_1, n_2, n_3, \cdots$, are the same. Let x be the number of figures in the period for $1/n$, where this time $n=2^c\times5^k\times n_1{}^t$. Here c and k are any integers each equal to or greater than 0, t is any integer greater than 0, and n_1 is a prime greater than 5 and less than 487. Let x_1 be the number of figures in the period for $1/n_1$. Then $x=x_1\times n_1{}^{t-1}$. For instance:

$\dfrac{1}{14440}=\dfrac{1}{2^3\times5\times19^2}\cdot$ The period generated by $\frac{1}{19}$ contains 18 figures. Hence $x=18\times19^{2-1}=342$, so $\frac{1}{14440}$ will have 342 figures in its period. The reason for excluding $n_1=3$ is that $\frac{1}{3}=.3333\cdots$ and $\frac{1}{3^2}=.1111\cdots$, each having only one figure in its period. There is no other exception until 487 is reached. Upsetting as it may seem it is nevertheless true that both $\frac{1}{487}$ and $\frac{1}{487^2}$ have 486 figures in their respective periods.

Finding the Period

The problem of finding the period of the repeating decimal for the fraction m/n is usually little more difficult than the problem, just considered, of finding the number of figures in the period. The most obvious method is to divide m by n, using long division, and continuing to add zeros to the dividend until a remainder is obtained which is either the same as m or the same as an earlier remainder. For instance, $8\div11$ gives immediately after the decimal point the figure 7, next 2, after which the remainder is 8, the same as the original dividend. Hence $\frac{8}{11}=.7272\cdots$. In dividing 7 by 12, the first figure after the decimal point would be 5, next 8, and then 3, after which the remainder would be 4, the same

as the remainder after the figure 8 was obtained. Hence $\frac{7}{12}=.583333\cdots$. To use this long division process for a fraction like $\frac{1}{487}$, which gives 486 figures in its period, the calculator would have to provide himself with a goodly supply of paper and much patience. For such fractions there are available methods that are easier to apply but not so easy to explain.

A quick method, nearly always usable, for finding the repeating period for $1/n$ will be explained. The first step is to divide 1 by n using long division, continuing the process until a considerable part, p_1, of the period has been found and until the remainder, r, is small, the smaller the better. Then set $1/n=p_1+r/n$. Multiply each side of this equation by r, thus obtaining $r/n=$ a new value p_2 plus a new fraction. Multiply each side of this equation by r, obtaining a third value p_3 plus a fraction. The decimal parts of p_1, p_2, p^3, $\cdots$ taken in succession will constitute the full period, p. The successive multiplications by r are continued until repetition is apparent, utilizing the information given in a previous section as to the possible number of figures in the period.

As an illustration let us apply the method to $\frac{1}{29}$. Long division gives the first of these equations and the others are obtained by successive multiplication of each side by 8.

$$\frac{1}{29}= \quad .03448\frac{8}{29}$$

$$\frac{8}{29}= \quad .27586\frac{6}{29}$$

$$\frac{8^2}{29} \text{ or } 2\frac{6}{29}= \quad 2.20689\frac{19}{29}$$

$$\frac{8^3}{29} \text{ or } 17\frac{19}{29}= \quad 17.65517\frac{7}{29}$$

$$\frac{8^4}{29} \text{ or } 141\frac{7}{29}= \quad 141.24137\frac{27}{29}$$

$$\frac{8^5}{29} \text{ or } 1129\frac{27}{29}=1129.93103\frac{13}{29}$$

Therefore, the period for

$$\frac{1}{29} \text{ is } 0344827586206896551724137931.$$

In actual practice the left side of each equation would be entirely omitted, as would also the numbers 2, 17, 141, and 1129 that precede the decimal point. In this abridged form the calculation for $\frac{1}{49}$ would be

$$.0204081632653\frac{3}{49}$$

$$.0612244897959\frac{9}{49}$$

$$.1836734693877\frac{27}{49}$$

$$.5510204081632\frac{32}{49}$$

Hence the period for

$$\frac{1}{49} \text{ is } 020408163265306122448979591836734693877551.$$

It might be interesting to apply this method to a really long repeating decimal. The following lines are the successive stages in the calculation of the period for $\frac{1}{487}$, the first line being obtained by long division and each of the others by multiplying the preceding row by 5.

$$.002053388090349075975359342915811088295687885\frac{5}{487}$$

$$.010266940451745379876796714579055441478439425\frac{25}{487}$$

$$.051334702258726899383983572895277207392197125\frac{125}{487}$$

$$.25667351129363449691991786447638603696098562\,6\ \frac{138}{487}$$

$$.28336755646817248459958932238193018480492813\,1\ \frac{203}{487}$$

$$.41683778234086242299794661190965092402464065\,7\ \frac{41}{487}$$

$$.08418891170431211498973305954825462012320328\,5\ \frac{205}{487}$$

$$.42094455852156057494866529774127310061601642\,7\ \frac{51}{487}$$

$$.10472279260780287474332648870636550308008213\,5\ \frac{255}{487}$$

$$.52361396303901437371663244353182751540041067\,7\ \frac{301}{487}$$

$$.61806981519507186858316221765913757\underline{7}002053388\ \frac{44}{487}$$

The period for $\frac{1}{487}$ is composed of the 486 figures in order up to and including the underscored 7 in the last line, and omitting the fractions at the ends of the lines. Each of the three results just given illustrates the complementary relationship between the two halves of a period and the rules governing the last figure of a period. These rules furnish a check on the accuracy of the work.

In a previous paragraph $\frac{126}{487}$ was mentioned as a fraction whose period would be hard to find by long division. Its period is now easy to obtain, since it will be the same as the one above for $\frac{1}{487}$ except that it will start at a different place in the sequence. The place at which to start can be found by dividing 126 by 487 to a few figures by long division, giving $.258726+$. By inspection it is apparent that the period for $\frac{126}{487}$ starts with the tenth figure in the third line.

The Converse Problem

Every repeating decimal is generated by a fraction, m/n, in which m and n are relatively prime integers. To find this fraction two methods are available. The first is based on the fact that a repeating decimal is either wholly, or in part, a converging geometric progression. Suppose the repeating decimal is $.148148\cdots$. This is equivalent to $\frac{148}{1000}+\frac{148}{1000000}+\cdots$.

The formula $S=\dfrac{a}{1-r}$ gives $S=\frac{148}{1000}\div(1-\frac{1}{1000})=\frac{148}{999}=\frac{4}{27}$. If some of the figures immediately after the decimal point do not belong to the period, only the repeating series forms a geometric progression. Thus, $.452727\cdots=\frac{45}{100}+(\frac{27}{10000}+\frac{27}{1000000}+\cdots)$. The sum of the geometric progression is $\frac{27}{10000}\div(1-\frac{1}{100})=\frac{3}{1100}$ Hence $.452727\cdots=\frac{45}{100}+\frac{3}{1100}=\frac{498}{1100}=\frac{249}{550}$.

The second method of finding the fraction, m/n, which will generate a given repeating decimal is based on the general rule, stated before, that if the period, p, has x figures in it, then 10^x-1 must be exactly divisible by n. It can be shown further that $\dfrac{m}{n}=\dfrac{p}{10^x-1}\cdot$ If the repeating decimal is $.216216\cdots$, we have $p=216$

and $x=3$. Then $\dfrac{m}{n}=\dfrac{216}{10^3-1}=\dfrac{216}{999}=\dfrac{8}{37}\cdot$

A repeating decimal like .8272727 $\cdots$ must be separated into two parts. Thus, $.8272727\cdots)=\frac{8}{10}+\frac{1}{10}(.272727\cdots)=\frac{8}{10}+\frac{1}{10}(\frac{27}{99})=\frac{8}{10}+\frac{3}{110}=\frac{91}{110}$.

Some Trick Problems

Three number tricks based on repeating decimals will be explained. The first one is old. Tell the members of a class to multiply 12345679 by 18. Watch the surprised looks on their faces when the answer comes out 222,222,222. Other multipliers may be used; to get all 3's multiply by 27, all 4's by 36, and so on up to 81. This trick is due to the fact that the repeating period for $\frac{1}{81}$ is 012345679.

The following trick has been worked out and used by the writer. The performer asks the members of a class each to do any division problem in which the dividend can be any prime number from 15 to 100 and the divisor any of the numbers 5, 7, 9, 11, or 13. He tells them to get the quotient to the first two decimal places, not however the closest second figure in the decimal. The person doing the trick asks each one in turn what quotient was obtained, and tells without any more information what dividend and divisor were used. It is possible to determine the divisor from the two decimal places given. If 5 is the divisor, the decimal will be an even figure followed by zero. If 7 is the divisor, the decimal will be two figures in order taken from 142857, the period for $\frac{1}{7}$. If 9 is the divisor, the decimal will contain two identical figures, such as .11, .22, .33, etc. If 11 is the divisor, the decimal will contain two figures whose sum is 9, such as .09, .18, .27, etc. If the decimal does not fit any of these specifications, the divisor must be 13. After the divisor has been found, it is a simple calculation to determine the dividend. With a little practice the performer can do the trick very quickly.

The third trick, also arranged by the writer, requires that the performer memorize the repeating period for $\frac{1}{17}$, which is 0588235294117647, and also $\frac{1}{3}$ of this, which is 196078431372549. The performer begins by claiming that multiples of 3, such as 3, 6, 9, 12, etc., can be easily multiplied by large numbers. Then he nonchalantly puts the second of the above numbers on the board, stating that he will attempt to multiply it mentally by any multiple of 3, below 50, named by the class. After a certain multiplifier has been agreed upon, the class is told to do the multiplication on paper so that they will know what is the correct product. Some members of the class may be detailed to watch that the performer does not do any cheating. The performer then mentally multiples $\frac{1}{3}$ of the selected number by the two last figures in the first number given above. The last two figures thus obtained will be the last two figures of the product, the rest of which will be the other numbers in sequence taken from the period for $\frac{1}{17}$. Suppose 42 is selected as the multiplier. When 14 is multiplified by 47, the last two figures thus obtained are 58. The easiest way to do this is to say 14×7 gives 98; put down 8 and carry 9; 14×4 gives 56; 9 to carry gives 65; put down 5. These are the last two figures of the desired product, the whole of which is, therefore, 8235294117647058. Since nobody is likely to have the slightest inkling of how the trick was done, the performer is usually credited with being a wizard at figures. Should the class express a wish to try it again with different numbers, the performer can beg off by saying that it is too hard work for him to do more than one.

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Curriculum Revision in Elementary Mathematics in Chicago

By J. T. JOHNSON, *Head*
Department of Mathematics, Chicago Normal College, Chicago, Ill.

WHEN the statement was made by the Superintendent in Chicago that formal arithmetic was not to be taught in grades one and two, a great cry went up from both teachers and lay people. Many primary teachers and principals thought it was a serious mistake to leave out formal arithmetic from these two grades. Various women's clubs made protests in their meetings against this postponement of arithmetic, as they thought, until the third grade.

We all know how much vicious transformation can be effected through hearsay and gossip. When the superintendent first made the statement the emphasis was upon the word *formal.* When the report reached the principals and teachers the emphasis had been shifted to the word *arithmetic* and when it reached the members of the women's clubs the word *formal* had disappeared and it was that no arithmetic was to be taught in grades one and two. Later, after many reports had been relayed and they had reached suburbs and towns outside the city limits the words *grades one and two* had been dropped in some places and arithmetic was to be dropped from the curriculum altogether. As evidence of this I have had a letter from the president of the school board in a small town in Illinois saying that since we (Chicago) had decided to drop arithmetic from the course of study, he would do likewise. The letter, addressed to the superintendent was referred to me for reply. I was glad to have the opportunity of telling this school board that we were making no eliminations in arithmetic but were adding more arithmetic of a different nature to the curriculum.

When in October of 1936 I was asked to write the new course of study in arithmetic for Chicago, it became my obligation to define the term, *formal* arithmetic. Our committee defined formal arithmetic to be arithmetic that included these three requirements; drill upon number combinations, definite time limits, and definite grade requirements. We had to reiterate this time after time because it seemed that teachers had the ineradicable notion that as soon as a teacher taught 2 plus 2 equals 4 she was teaching formal arithmetic. If that is the commonly accepted meaning of formal arithmetic we have modified the meaning in Chicago to fit the aforestated definition.

The learning of numbers and mathematical concepts begins very early in the child's life. It begins much earlier than the learning of reading. Shortly after the age of two the child begins to form concepts of numbers in connection with his counting of objects and in recognizing meanings of cardinal numbers. He does not begin to learn to read, however, until after the age of 5 or 6. The new course of study in arithmetic in Chicago recognizes this fact and hence does not leave out arithmetic in the first and second grades nor even in the kindergarten. I like to call the kind of arithmetic to be taught in the kindergarten and grades one and two, social arithmetic, which it really is, rather than informal or incidental. *Social* is a stronger term and has a more fitting connotation. The new course in the first and second grades gives detailed outlines and even typical class room lessons for teaching this social arithmetic in connection with the following departments; reading, social studies, elementary science, art, music, handwork, health work, games and play. This socializing of number work in different fields is intended to give it meaning so that when later in the third grade the formal aspect of number is taken up it will then be full of meaning acquired throughout the first two grades in a de-

velopment there of social situations rich in number meanings.

I am fully aware of the danger incident to the announcement of no formal arithmetic in the first two grades. Some teachers will think that the arithmetic in these two grades is unimportant and leave it out. Others who see its importance will not know how to teach this social arithmetic. That is why, all the more, we should emphasize its importance and provide in courses of study and number readers explicit information and guidance on the nature of arithmetic teaching in these grades. That is why we have in Chicago in our set up of courses for the new four-year Normal College provided for a special course in the arithmetic of the kindergarten and grades one and two. A special course is also given in this work in our summer school for teachers already in the system.

The recent investigations of what number knowledge is possessed by six year old children entering grade one is in danger of being misinterpreted. Let us not forget that what number knowledge the child may have at the age of six has been the slow acquirement of four years. Because of the abstract nature of number its acquirement is a slow process. It took the race thousands of years to develop much less than what is required of a third grade pupil to-day. This slow development of number concepts is highly significant to elementary mathematical education. Makers of courses of study and writers of arithmetic texts have not always fully appreciated its importance. Too much has been required of the pupil during the early years of his school life.

In teaching reading, carefully graded vocabularies are made to fit the stages of the child's growth. There are now, I am told, graded books and even newspapers whose vocabularies are limited to 900 words.

In teaching spelling, special graded lists are provided and the child is not required to spell a word until he knows its meaning and can use it in a sentence.

Why would not this be good practice in arithmetic and not have a pupil do an example until he knows its meaning and can use it in a verbal problem?

In examining some of the leading courses of study of the last decade we find that by the end of the third grade the pupil is required to know the numbers from 1 to 999 and in addition to this the 390 fundamental number combinations. This constitutes about 95% of the arithmetic vocabulary of the average adult. And let us not forget that a number vocabulary is more difficult to acquire than a word vocabulary. The concept *cat* is more easily learned than the concept *five*. There is more similarity between a black cat and a white cat than between 5 pennies and 5 days. The generalization that 3 plus 2 equals 5 applies to all objects and things in the universe. No such generalization in words alone is required of the third grade child.

In an analysis of the fundamental skills of arithmetic we find that in the process of addition the two most difficult skills—the one of carrying from one column to the next and the other of holding a number in mind while another is added to it—are both required to be mastered by this third grade child. What more is there to be learned in addition? Not very much—largely more of the same thing.

In subtraction we require borrowing so-called and zero difficulties besides the 100 subtraction facts.

In multiplication the 100 facts and two-place multipliers requiring placement of the partial products is required.

In division we ask this third grader to know the 90 facts and some two-place division involving carrying.

In terms of new learnings these combinations and skills which we require to be learned by the end of the third grade constitute about 85% of all the new work required in the fundamentals of arithmetic through decimals. When we stop

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In terms of new learnings these combinations and skills which we require to be learned by the end of the third grade constitute about 85% of all the new work required in the fundamentals of arithmetic through decimals. When we stop

a moment to reflect on this does it not seem that our program is a little unbalanced, being overloaded at the lower end, at the place where the learner is also immature.

If we make a better distribution of the learning of these facts and skills so that the more difficult ones are not demanded before the child is able to master them and retain them it will mean moving up some of the topics such as the difficult phases of division and common fractions. There is no loss in economy of time involved in so doing because at a later age the child can learn much more in the same time as at an earlier age. The final outcome at the end of the eighth grade is the same.

It was the policy in the new course to spread the development of the main topics of arithmetic over a greater range of grades for better learning and more permanent retention. For example common fractions is begun in the first grade with the simple meaning of $\frac{1}{2}$ and $\frac{1}{4}$ and continued in a graded development through the seventh grade. Division is begun in the fourth grade and continued through the seventh grade before completion. Decimals is begun as decimals in the fifth grade and continued through the seventh grade before the difficult parts of division by decimals is completed. In the old course of study decimals was begun in the sixth grade and completed in the same grade. Whereas in the traditional course decimals have always been treated about a year later than common fractions, in our course they are begun together in the fifth grade. We have thus moved down the beginning of decimals one year. This causes no difficulty for we know that addition and subtraction of decimals are easier than the same operations in common fractions. The fact that common fractions were invented some 3000 years before decimals is no guarantee that they are easier to learn. By having them taught in close proximity to decimals the one can be used to strengthen the meaning of and prove the processes in the other and pupils may have the opportunity to actually compare the relative advantages of the two.

In distributing the various skills of each topic throughout the grades we were guided by results of tests given throughout the city for a number of years. Low test results in long division, division by decimals (not division of decimals) and operations with mixed numbers placed these topics later in the course. The findings of the committee of seven corroborated our test results and thus strengthened the reason for moving up many of the topics.

Something should be said in connection with what the committee of seven has done. Theirs is the only investigation on a large scale that has attempted an experiment on grade placement. It is the best we have at this time. It is not faultless but if taken at what it claims for itself it is valuable as a point of departure for future investigations. One of the weaknesses lies in the fact that it did not have at its disposal the skills of arithmetic sufficiently well analyzed for teaching purposes. The skills in common fractions and division for example are so many and so varied in difficulty that they should have been tested at more levels than were tested. If this had been done perhaps the grade placements of many very simple aspects of a subject would have been lowered. This remains to be done. The difficult aspects and those that were tested show decided lack of mastery and retention at the grade levels where they are usually taught.

Mr. Grossnickle has done some experimenting, I believe, in grade placement and has shown that division with two-place divisors can be mastered in the fourth grade. I think he will find that division by two-place divisors such as 21, 32, 41 is more easily learned than division by 7, 8, and 9. We want more of this kind of experimentation. By intensive teaching a bright child can master mechanically quite intricate processes. But if he forgets

these processes through disuse the question is raised as to the wisdom of teaching him a process before it is needed. Here again we need more research. Two questions need answers. One is, shall a process in arithmetic which has been found useful in the social activities of a certain grade be taught under high pressure even if it is very difficult to master? On the other hand, shall a skill that has been found easy of mastery at a certain grade level be taught there even though there be no need for it in the natural life of the child of that grade?

Other questions more pressing than these will have to be solved and I believe a research body such as the recently organized National Committee on Arithmetic will do it.

In the meantime, Chicago is going to play safe in teaching arithmetic by placing those topics that are difficult at a place more commensurate with their mastery for two very definite reasons, first, because by so doing we can more adequately secure meanings for what is taught and second, because of the effect on the arithmetic morale of the pupils. The wholesome effect on the child when he succeeds at his tasks cannot be overestimated. A consciousness of continual failure in not attaining a goal set for him is demoralizing to any child. The effect of moving up topics about a year gives more children opportunities to master as they go along and thus establish confidence in themselves which is so essential to growth.

To summarize, this is a dual world and there are two kinds of arithmetic as there are two kinds of everything else. We have social arithmetic and pure or formal arithmetic and they should interplay in a forever mutually reenforcing and enlightening way. We believe in continuing the social arithmetic in the kindergarten for it has already begun with the child. We believe in making it a definite part of the education in the first and second grade. In this social program number combinations, any of them, unavoidably enter in, even fractions and United States money. But as long as we do not require drill with definite time limits and grade goals it will be enjoyed by the pupils and they will not develop inferiority complexes with respect to arithmetic.

It is our aim to make arithmetic an enjoyable subject free from so many failures and to make the conditions favorable for a pupil to go through the elementary school with his arithmetic sense undebauched and ready to enter high school with a feeling that mathematics is a respected, important and delectable subject.

Notice to Subscribers

NOTICES have recently been sent out to all subscribers whose subscriptions expired in May asking them to send in their renewals at once so as to save both them and *The Mathematics Teacher* unnecessary trouble and expense. For the sake of clearness, we must emphasize the fact that the magazine does not appear in June, July, August or September and hence a subscription which begins with the October issue expires with the eighth issue—May—and similarly for a subscription beginning with any other month. Only eight issues are given for $2.00.

Most of our subscribers have sent in their renewals promptly and this is a great help to us. We would urge all subscribers, who have not done so, to read the material on the inside front cover of this issue. Some important and useful material is given there.—The Editor.

"If-Then" in Plane Geometry

By Harry Sitomer
New Utrecht High School, Brooklyn, N. Y.

According to the National Committee report, the principal purposes for teaching plane demonstrative geometry are:

1. To exercise powers of spatial imagination,
2. To familiarize the student with the great basic propositions and their applications,
3. To develop an understanding and appreciation of deductive proof, and the ability to use this method,
4. To form habits of precise and succinct statement, of logical organization of ideas and of logical memory.

Teachers and students fail more often with respect to the third and fourth aims. Perhaps the reasons for this failure may be grouped under three classifications:

1. Our formal examinations test largely for a knowledge of basic propositions and their applications, while only a minor part test the student's ability to form deductive proofs.
2. It is more difficult to develop an understanding of deductive proof since many fundamental concepts must be mastered first. Our minds and emotions are tuned to a fast pace and we tend to "jump to conclusions" in spite of the necessity of careful and painstaking scrutiny of idea-relations required in deductive proof.
3. The procedure of demonstration is uncommonly beset with too many pitfalls for the unwary and uninitiated.

To the beginner there are many seemingly unrelated ideas and rules which seem unnecessary since he already comes to class with a notion of "logic." This notion often is vague and means simply "something reasonable" or "something correct" or "something true." Every one of these chaotic notions needs refinement and in many cases complete nullification before the student can appreciate demonstrative proof.

The common types of mistakes may be recognized by every experienced teacher of geometry to be the following:

1. Omission of statements,
2. Inclusion of irrelevant statements,
3. A disregard for a correct order of statements,
4. The use of reasons not yet established,
5. Reasoning in a circle—i.e., the use of the proposition in question as a reason in its proof,
6. The confusion of definitions with theorems,
7. The confusion of the hypothesis with conclusions,
8. The confusion of a statement with its converse and inverse.

The frequency of these mistakes convinces the writer that although students pass examinations, they do *not* appreciate deductive proof and therefore do not and cannot develop "habits of succinct and logical statement or logical memory."

The problem facing most teachers of plane geometry is then "What can they do to teach this appreciation?" Furthermore "Is it possible to determine objectively when the student has this appreciation?" Involved in this problem is also that of teaching the student to criticize demonstrations of fellow students or justify his own proof to the most doubting of his class.

The first step in the solution of this pedagogic problem is to make clear to the student what deductive proof means. Ideas of truth, self-evident or otherwise, authorities (text and teacher), common sense, etc., must *not* be taken as synonyms for logical proof. The definition of logical proof must contain within it the means of detecting soundness or falseness in proofs. It must contain within it an instrument which identifies deductive proof. This is the crux of the matter and if answered, will contain the nucleus for the

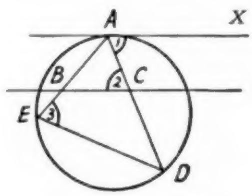
discovery of a suitable pedagogical procedure.

The answer has been inherent in the work of those mathematicians who have pried into symbolic logic and investigated propositions and propositional functions. The writer submits that the "if-then" form ought to be used for *all* axioms, definitions, and theorems, no matter how simple they may be. As illustrations, let us consider:

Axiom:	If a quantity is halved, then the halves are =.
Definition:	If a $\triangle$ has two = sides, then it is called an isos. $\triangle$.
Theorem:	If $\measuredangle$'s are vertical, then they are =.

This procedure has the immediate advantage of making clear the difference between hypotheses and conclusions. Also the two-fold uses of definitions may be pointed out since it is of the nature of definitions to interchange the "if" and "then" clauses with no violation of validity.

Let us now consider a proof when all reasons (except "given" and "identity")



are written in the "if-then" form:

Given: AX is a tangent at A
$BC \parallel AX$
To prove: $AB \cdot AE = AC \cdot AD$

1. $BC \parallel AX$.	1. Given.
2. $\angle 1 = \angle 2$.	2. If lines are $\parallel$, then a trans. makes alt. int. $<s=$.
3. AX is a tangent at A.	3. Given.
4. $\angle 1 = \frac{1}{2}\widehat{AD}$.	4. If an $\measuredangle$ is formed by a tangent and a chord from the point of contact, then it is measured by half its int. arc.
5. $\angle 3 = \frac{1}{2}\widehat{AD}$.	5. If an $\measuredangle$ is an inscribed $\measuredangle$, then it is measured by half its int. arc.
6. $\angle 1 = \angle 3$.	6. If two quant. each = a third, then they are = to each other.
7. $\angle 2 = \angle 3$.	7. If two quant. each = a third, then they are = to each other.
8. $\angle A = \angle A$.	8. Identity.
9. $\triangle ABC \sim \triangle ADE$.	9. If $AA = AA$, then $\triangle$s are $\sim$.
10. $\dfrac{AB}{AD} = \dfrac{AC}{AE}$.	10. If $\triangle$s are $\sim$, then corr. sides have = ratios.
11. $\therefore AD \cdot AE = AC \cdot AD$.	11. If ratios are =, then prod. of means = prod. of extremes.

The reader will notice that the "if" clause of reason 11 is the same as the "then" clause of reason 10; also that the "if" clause of 10 is the same as the "then" clause of 9. In 9 there are two parts in the "if" clause. The first, "$A = A$," is the same as is implied in "identity" of 8, and the second "$A = A$" is found in the "then" clause of 7. Likewise, the "if" clause of 7 has two parts and can be traced back to the "then" clause of 6 and the "then" clause of 2. This "tracing back" process will involve every "if" and "then" clause in our reasons until we reach "identity," "given," or an "if" clause such as that in 5 which is implied as "given" if we refer to the diagram. This use of "then" clauses acting as subsequent "if" clauses is the essential core of demonstrative proof. Every part now is seen to be part of a purposeful whole which aids in showing how the original hypothesis eventually implies the desired conclusion. This relationship may now be summarized symbolically as follows:

Let h_n and C_n represent the "if" (hy-

pothesis) and "then" (conclusion) clauses respectively in the nth reason, and let $h_n \equiv C_m$ mean that the "if" clause of the nth reason is the same as the "then" clause of the mth reason. Let h_{na} represent the a*th* part of the "if" clause in the nth reason. ($n=1, 2, \cdots, a=$ a, b, c, $\cdots$). It may now be seen that

$$h_{11} \equiv C_{10},\ h_{10} \equiv C_9 \begin{cases} h_{9_a} \equiv \text{Identity (8)} \\ h_{9_b} \equiv C_7 \begin{cases} h_{7_a} \equiv C_6 \begin{cases} h_{6_a} \equiv C_5,\ h_5 \equiv \text{Given (implied)} \\ h_{6_b} \equiv C_4,\ h_4 \equiv \text{Given (3)} \end{cases} \\ h_{7_b} \equiv C_2,\ h_2 \equiv \text{Given (1)} \end{cases} \end{cases}$$

(note: I would *not* suggest that students learn this notation).

When the student appreciates the relationship of the parts in the reasons used in a deductive proof, certain advantages should be obvious to the teacher. He can now show why statements are misplaced in order, are missing, or are not needed. In each case this retracing process will break down. Exercises, in which students do this tracing may now be done, with the purpose of given them the experience of observing and appreciating this linkage of a series of statements in the "if-then" form. The teacher may also take the further step of showing a definite relationship between a reason and the statement it justifies; namely the relationship of specific versus general. The student should appreciate that the reasons may refer to any situation which satisfies the condition stated in the "if" clause and that this condition has been shown to exist, previously, while the "then" clause is a general statement of the condition stated in the same numbered statement.

With this background the following definition may be enunciated. "Validity in demonstrative proof is that property of an ordered series of established or assumed statements in the 'if-then' form which enables us to trace the last 'if' clause through previous 'then' clauses and their respective 'if' clauses to the hypothesis, expressed or implied (I consider 'identity' as an implied hypothesis), and in which every 'then' clause is a general statement of a specific statement mentioned."

This definition should not be considered as an end in itself, but rather as a formula which will guide the teacher in devising teaching procedures which will enable the student to appreciate and understand the concepts involved in deductive or demonstrative proof. These concepts will include the following:

1. The appreciation of the difference between a statement of fact and a statement of implication.
2. The appreciation of the difference between the hypothesis and the conclusion in a statement of implication.
3. The appreciation of the relation between a theorem—its converse, inverse and contrapositive.
4. The appreciation of the meaning of necessary and sufficient conditions.
5. The appreciation of the difference in procedure between direct and indirect proofs.
6. The ability to recognize validity in demonstrations.
7. The ability to recognize errors in demonstrations.
8. The appreciation of the sequence of theorems.
9. The ability to apply these concepts to non-mathematical materials.
10. The ability to apply these concepts to algebraic processes.
11. The appreciation of the place of assumptions, undefined elements, definitions, and theorems in a logical system.
12. The ability to use analysis.

The writer believes that the "if-then" form and the definition of deductive proof given above will aid considerably in teaching all of these abilities and concepts. This was indicated, or is obvious to every teacher of plane geometry in the cases of items 2, 3, 4, 6, and 7. Let us consider

items 1 and 9. The following exercise may be assigned to students:

Select any short editorial and do the following after reading it carefully:

1. Write a statement of what the writer of the editorial assumes as "given" and what you think he is trying to prove.
2. Rewrite the editorial tabulating the statements and reasons as you would in the proof of a geometric theorem. Wherever necessary supply statements or reasons implied by the writer.

When reports are made criticisms and appraisals may then be made by students. The writer has had some experience with this type of lesson and on that basis reports highly satisfactory results.

As for item 5 three possible procedures are possible, namely those implied in:

1. If two conclusions are possible for a given hypothesis so that both cannot be true or both false, and if one leads to a contradiction of the hypothesis, then the other is established.
2. If $\bar{q} \to \bar{p}$, then $p \to q$ (the method of contraposition so ably presented by Dr. Nathan Lazar, as was the following)
3. If (p and $\bar{q}$) are inconsistent then $p \to q$.

The point made here is that putting these ideas in the "if-then" form should make the whole procedure clearer to the student since the "if" clause indicates the conditions which must first be established before the student can make the desired conclusion.

Teaching techniques can also be devised on the basis of the "if-then" procedure by any competent teacher to achieve those other aims not here discussed. The ability to analyze can be taught now by the "then-if" procedure, in which the "then" clause is followed by the "if" clause.

In closing, it may be said that while Dr. Nathan Lazar's thesis has clearly shown how converses and contrapositives can be used in the class-room as classifiers and generators of theorems, so a proper use of the "if-then" form and the definition given above make the concepts inherent in deductive proof clearer to students. This clarity should result in an improvement in the abilities of students to use deductive reasoning.

A Reaction to "Let's Check the Hypothesis" by Edwin G. Olds in THE MATHEMATICS TEACHER, December 1937

Once upon a time some very bright men were seeking mental vitamins, only they didn't call them such. They thought they discovered in mathematics the one necessary to the development of logical thinking.

Now it has never been proved, so far as I know, that logical thinking is necessary in human life. In fact, a great many people seem to get on extremely well without having, apparently, any use for such procedure. Yet, just as nowadays a child must eat spinach and carrots if he is to be properly nourished, whether he likes them or not, so, in that once-upon-a-time, children who were to become educated had to partake mentally of mathematics, willy nilly.

And so it happened that many grandparents and parents now living, after much forceful feeding and in spite of many cases of mental indigestion, came to have an acquired taste for this mental green, or at least they so pretend. And now they wish their offspring to have mentalities nourished by the same food,—just as this generation of youth will in a few years insist that the rising generation partake of spinach and carrots.

It is entirely within the realm of possibility, even of probability, that when youth shall have arrived at the point of taking spinach and carrots as a matter of course, some one will discover that they aren't essential at all,—just as one has been hearing during the past score of years that mathematics isn't essential to education; that, on the contrary, in the opinion of certain folk who are sure they know, mathematics is of very little worth.

Notwithstanding, spinach and carrots, with their rich and appetizing colors, will brighten many a frugal meal and grace many a festive board in the future. And mathematics, with its rich history of three thousand years and more; mathematics, with its charm for those who do think logically; mathematics, with its ever increasing usefulness as the basis of scientific thought and discovery; mathematics, which may justly share with speech the honor of raising man from the brute creation; mathematics will continue eternally to tickle the intellectual palate of youth.—CLYDE H. LADY.

Trends and Policies in Arithmetic Curricula

By B. R. Buckingham
Editorial Department Ginn and Company, Boston, Mass.

During the past half dozen years events have been moving pretty fast in the field of arithmetic. I may be the victim of an illusion—an illusion arising from concentration of attention upon a matter of considerable personal interest, yet I have other personal interests, and to those interests I have often devoted more attention than I have ever given to arithmetic. It is my deliberate judgment, to be taken for what it is worth, that an unusually rapid change is taking place in the subject-matter of arithmetic and in the objectives entertained by students and teachers of the subject. To document this would perhaps be tedious. One may, however, point to one large general trend which includes within itself many subordinate trends and implies still others as a natural consequence.

I refer to the almost complete transformation in the accepted psychology of arithmetic. Ten years ago all the leaders in this field were atomists who pinned their faith to the doctrine of definite responses to definite stimuli, and to the virtues of minute analyses of subject-matter. They exalted specific skills which they sought to develop by drill according to the law of exercise. They believed that each big skill was the sum of little skills, and that mastery of a field of subject-matter was the total of subordinate masteries in constituent areas.

Today, with almost the same universality, students of arithmetic adhere to some type of Gestalt psychology. They are more interested in wholes than in parts, more convinced of the efficacy of insight than of the sufficiency of repetition, more concerned with understanding than with skill, and more than ever reverent of the deep mysteries of individual differences.

Under these circumstances, an about-face is everywhere evident among students of arithmetic. Some who were particularly able exponents of drill theories have nimbly turned their coats to appear in the camp of the configurationists. I have nothing but praise for their lack of consistency. A few, it is true, while taking the new ground, now attempt to convince their readers or hearers that they never took any other ground. But for the most part, the writers on arithmetic of the late twenties have silently forsaken their earlier tenets and have accepted without loss of dignity the doctrine of the late thirties.

We now realize that this inveterate analysis—this teaching of small pieces without reference to larger wholes—was never really fundamental in itself. This procedure was honestly adopted as a means to an all-important end, the end being a developmental program which should take its pace and seek its objectives in accordance with the maturity of the growing child. Originally, therefore, analysis was subordinate to a larger purpose. Its abuse came when it was made an end in itself.

Those who repudiate analysis in anything like the thoroughgoing fashion which used to be indulged in, must not at the same time repudiate the purpose which analysis was intended, however mistakenly, to serve. The new doctrine must offer a substitute for the developmental or graded approach which the analyst had in mind. I think I see what the new doctrine is going to offer as such a substitute. Constrained as it is, and should be, to the presentation from the beginning of whole thoughts, it must offer these whole thoughts in such a manner as to accom-

* An address delivered at the meeting of the National Council of Teachers of Mathematics, New York City, June 27, 1938.

plish two purposes: first, it must appeal to the learner at his current stage; secondly, it must be true, in the sense that it will not have to be contradicted at a later stage of the learner's development.

This makes peculiar demands upon the resourcefulness of curriculum-makers and teachers. In the primary field it means that a process will be initially presented as a process complete in itself and not as part of a process artificially derived from the whole. This is not to say that analysis may not follow; but it is a far cry from a method which offers analyzed parts at the beginning to a method which offers wholes at the beginning and subsequently analyzes them into parts seen in relation to the whole.

In securing a developmental approach—an approach, in other words, which grows with the growth of the child—the relative immaturity of the learner is recognized by tolerating and even encouraging types of procedure which, under the older psychology, were always avoided. In learning the number combinations, for example, the child is permitted to count. In column addition he is permitted to write the carried number. In subtraction he is permitted to write the altered digits when they have to be changed. In dividing by a one-place number he uses the long form. All these procedures were formerly taboo. They are coming to be recognized as a valid adjustment to the immaturity of the child.

An opprobrious term has been applied to these procedures. The children are said to be using *crutches*. Whether or not we keep this term is of no consequence, but the practice which it thus labels is assuming considerable importance. The tendency in the development of the curriculum today is to recognize the importance of crutches, to permit their use when needed, and to encourage children to pass on, as soon as their growth allows, to more mature, more facile, and more satisfactory ways. If the child clings to his crutches, it probably means that he cannot get along without them; and I submit that it is far better for him to walk with crutches than not to walk at all.

I note another tendency to secure within the framework of the present psychology a developmental procedure. It seems to me that a new and fruitful meaning is being given to an old concept, namely, the spiral treatment of subject-matter. There is a tendency to introduce in early stages the rudimentary forms of most of the topics which afterwards receive progressively more adequate treatment. The progressive character of this treatment, marching *pari passu* with the growth of the child, demands a spiralizing of instruction. Observe that the condition which I am describing requires that in the lower grades a number of large topics be begun more or less at the same time. Progress in all these topics is demanded and is essentially simultaneous.

In the primary grades, for example, the child is developing simultaneously his ideas of numbers less than ten, his ideas of teens numbers, and his ideas of all numbers up to 100. In many schools he is also developing at the same time his ideas of fractions, of money numbers, of telling time, of units of measure, and so on. Here is foreshadowed much of what will afterwards become more explicit in the middle and upper grades. In respect to any of these topics no child ever learns completely at his first experience. If his progress is to be graphically represented by a group of lines perpendicular to the plane at which he finds himself at the moment, then the only mathematical figure which can be made progressively to intersect at higher and higher points this family of verticals, neglecting none and coming around periodically to each, is the spiral. The inclined plane, or ramp, cannot do it because it cannot change its direction.

I have been interested for some years in the place which arithmetic has been assuming in the elementary curriculum. I do not mean the placement of particular topics, but the place of the subject itself.

Just now it seems to me that there are two tendencies. According to one of them, the estimation in which arithmetic is held—its place in elementary education—has been low and has been becoming lower. In the teens of the present century there was a passion for "minimum essentials" and you will remember that two or three Yearbooks of the National Society for the Study of Education were devoted to this topic. The idea of minimum essentials was applied to arithmetic with great vigor. The subject undoubtedly needed pruning, and it certainly *was* pruned. In the process of deciding what should be discarded and what should be retained, the principle of frequency of use was considered especially valuable. After a while we obtained an arithmetic course minus a great many of the trappings, and minus some of the really vital principles, which had hitherto characterized it. Yet the subject continued to be the most difficult in the curriculum, more children continued to fail in it than in any other subject, and dissatisfaction continued to prevail.

Clearly therefore the pruning of the subject had failed to serve the expected purpose. Something else was needed; and as investigators began to cast about for that something else, and as certain long-accepted principles of the psychology of learning began to give way, it became increasingly evident that what arithmetic was suffering from was not so much an over-loaded course as a course in which nobody was interested and which nobody understood. The crassly utilitarian point of view proved to be insufficient because the residual course, whatever might be left of it, was external in its source, superficial in its purpose, and unappealing in its method. Under those circumstances the place in the curriculum of the subject called arithmetic not only was low but deserved to be low.

The rescue squad came none too soon. The principle that arithmetic should contribute to life and to the needs of life arrived on the scene after a certain flight from arithmetic had already started. This flight, some of us believe, must be arested; and we think there is a good chance that it will be arrested.

Meanwhile the nature of this flight is worth a moment's consideration. As I look upon it, there are two kinds of retreat: one involves the postponement of the entire subject to later grades, the other involves the postponement of particular topics to later grades.

Those who retreat the first way begin arithmetic in the third grade, or perhaps in the fifth. One enthusiastic soul declares that he begins arithmetic in the seventh grade. These people maintain that, in spite of the postponement of the subject, their pupils learn as much arithmetic as those who begin the subject from two to six years earlier. Aside from the fact that the proof of this statement rests upon very dubious procedural grounds, it is perhaps enough for our present purpose to dismiss the matter with the statement that the postponement of arithmetic could hardly damage results so meager as were being secured under the old plan. In other words, a sterile type of arithmetic may produce such unsatisfactory results that it may make little difference when the subject is begun since it will be almost equally meaningless at any time.

Perhaps more should be said as to the flight which manifests itself in pushing up into higher grades the work which has hitherto been regarded as suitable for the lower grades. How far this has proceeded I can illustrate by reference to the report of a new course of study in one of our progressive city systems. This course, according to the report, attains its objective by "redistributing some of the previously included content through the upper-elementary and junior-high-school grades." The result is rather startling. In the low-third grade children are first taught the addition and subtraction facts through 10, despite the fact that most of them knew these facts when they entered the first grade two years before. In the high-third

grade the rest of the addition and subtraction facts are taught. In neither half of the grade is attention seemingly paid to number concepts, or to a consideration of the *idea of 10* in our number system. Children enter the fourth grade with no notion of either multiplication or division, and not until the second half of the fourth year are they taught even the easiest number facts in those operations. The 100 multiplication facts and the 90 division facts are not completed until the middle of grade five, and in that grade compound multiplication of integers is first begun.

Division by a one-place number, using the long form, is the chief work of the upper fifth grade. No work is done in fractions until the sixth grade when their meaning is first developed. In the first half of this grade addition and subtraction of fractions are taught while, strange to say, multiplication and division of fractions are pushed up so far that they pass out of the elementary course altogether. In the upper half of the sixth grade division of integers is completed by introducing divisors of two and three digits. Thus what we ordinarily mean by long division barely escapes being a secondary-school subject. In this modern curriculum, the junior-high-school teacher must teach as new topics all about decimals, all about multiplying and dividing fractions, and all about denominate numbers and percentage.

One practical criticism of this re-organized curriculum is that it overloads the junior-high-school course. But more serious than that is the fact that this curriculum, even considered (as I am forced, in the interest of brevity, to do) merely on basis of subject-matter classifications, fails dismally to keep up with the growth of the child. Miss Polkinghorne has shown us that the child already talks about small fractions and uses them in the first grade. Yet in this course he begins his work with fractions in the sixth grade. By virtue of postponing the multiplication facts until the fifth grade, the child of ten is unable to figure the cost, or obtain the amount of material frequently required in his school projects and in his out-of-school affairs. When two-place divisors are not reached until the high-sixth grade, a dozen or fifteen children cannot, at an age when they will want to do so, figure their share of the expenses of an excursion or a hike or a class play.

The point I am trying to make with you is that, in terms of topics—and I know how inadequate such terms are—neither the postponement nor the attenuation of the program in arithmetic will meet the needs of the modern school. Some of this is desirable but arithmetic must first of all be penetrated by a new spirit, a spirit in virtue of which it will be thought of as something to be desired and cherished, not something to be avoided and put off.

The arithmetic from which so many are attempting to escape is not the arithmetic which the better type of students of the subject are now offering for the consideration of the schools. It is not, for example, the type of arithmetic which your Arithmetic Committee under Doctor Morton's chairmanship is contemplating. On the contrary, the arithmetic which so many are seeking to avoid is a type which has created a great deal of trouble and has produced, I am sorry to say, scant results for the effort involved. You have only to look about you in order to see the pitiful results of the course we have been pursuing. Do not turn your attention merely to people whom we call uneducated. Quite generally it is the uneducated people who do the best quantitative thinking. They acquired this power through practical experience, an accomplishment which ought to offer suggestions for the conduct of the arithmetic work of our schools.

No, it is among the first-class products of our school system that the ineffective results of our arithmetic teaching are especially apparent. You find evidences of it among graduate students dealing with research problems. They figure furiously

and bedeck their theses with tables and graphs, but they do not know what the figures mean. They can add and subtract and multiply and divide; they can get medians and will do so endlessly; but their results suggest nothing to them. They have little in the way of interpretation to offer. All they possess is a certain barren facility in the manipulation of numbers. They are the product of our teaching of arithmetic. They can count but they cannot think.

Now, it is precisely that our pupils and our citizenry may *think*, that the modern curriculum in arithmetic, as well as in other subjects, is being organized. The desired outcomes are phrased in new words. The reaction of the pupil is judged according to new standards. In contrast with the program which the modern curriculum seeks to supplant, the right answer is often unsatisfactory and the wrong answer may under certain circumstances be highly commendable. There is coming to be a general appreciation that the way in which the child reaches a conclusion is often more important than its formal correctness. This is only apparently a failure to prize accuracy in computation. Such accuracy is indeed indispensable, but it has no value for its own sake. No one computes, with or without accuracy, except for a purpose to which computation will contribute. Moreover accuracy in computation, when sought by the early imposition of adult mechanical facility, is a mere jargon without sense, a jargon quickly forgotten when the surface facility and physical nimbleness cease to be practiced. It is not enough that children should perform with flawless accuracy the combinations and processes. It is true that these facts must be learned and learned automatically, but we may defeat our ends if we demand automatic mastery before the learning organism can be expected to produce it. Moreover knowledge of facts as such does not constitute a sufficient objective in the new curriculum. Such an objective merely recognizes that facts *exist*. It fails to recognize that they have *meaning* and *value*. Unlike the knowledge of facts as facts, the value of these materials is *personal*.

With this I come to one of the great underlying characteristics of the modern curriculum, whether it be in arithmetic or in other subjects: the personalized nature of it. We do not hear so much today as we used to hear about mass instruction and mass learning. Perhaps the reason for this is an appreciation of their absurd contradiction in terms. Learning is as personal as your toothbrush. It cannot be exchanged for someone else's learning. Accordingly there is no such thing as mass learning, and no such thing as mass instruction. The curriculum today, in its more subtle adaptation to individual differences is frankly facing the necessity of the personal appeal in all our dealings with pupils. It is doing so not on the basis of preaching, not on the basis of lecturing about what children should be interested in, but on the basis of a more direct and intimate individual appeal. It is not enough to assure a class of pupils that they are going to have fun with this next topic. We must contrive to be convincing to each pupil, for each is concerned only with his own fun, and an activity which does not interest him personally has no interest at all.

This is not an idle fancy, nor is it a mere theory. There is abundant and respectable research which justifies us in saying that value—personal value—is assuming high importance in respect to the curriculum. For example, we have been told that children should check their work, that they should adopt the habit of checking everything they do. Do you do so? Does anybody do so? I mean universally? As a matter of fact, do we not check when we feel some uncertainty, or when the issues at stake are so important that a mistake would be serious? Grossnickle, in a recent article, indicates that checking, quite contrary to the usual notion, has no value in producing or maintaining accuracy. His

findings lead him to advise us, with respect to checking, as follows: Teach it when it has admitted value, when its use and significance can be understood. This admitted value and this understanding of use and significance do not come to all the children of a given class at once. Some may have them at a certain moment; others may not. Thus these matters of real importance are found to be individual, intimate, and personal.

There are many things which I should like to say apropos of this topic, but they must be deferred to another occasion. Before I conclude I should like to plead the cause of the children of the middle grades. In arithmetic the course for the middle grades has for a long time been especially drab and uninteresting. That this is a fact with other subjects is not my story to tell. That it is due to certain general conditions having no direct reference to the curriculum is still another story. But with respect to the course in arithmetic, I wish to suggest that we have made many improvements in primary mathematics, and not a few in secondary-school mathematics, but in respect to the course for Grades Four, Five, and Six we have failed so far to introduce either the solid substance of research or the quickening leaven of our enthusiasm.

In mathematics the child somewhere in the third or fourth grade attains knowledge of about 1500 facts to which, let us hope, he attaches some richness of meaning. It is a curious coincidence that at about the same time in his educational progress he has acquired in reading about the same number of facts—what we call his word knowledge. With this equipment in reading we hold that he has passed through the first stages, and that having attained a modicum of mastery of the mechanics of reading he is ready to read with greater accuracy of comprehension and depth of understanding—to read more widely for both pleasure and information and thereby to enrich and extend his experience and vision as much as possible.

I think it is not otherwise in mathematics. At about this level the well-taught child should be ready to pursue his study of the subject with different objectives and a more humanistic outlook. I predict that if the trend in the course of study in mathematics follows the path that lies most evidently before it, it will exalt the curriculum of the middle grades during the next ten years as it has exalted that of the primary grades during the past ten years.

Finally I see a tendency in respect to mathematics in the elementary school is to seek new strands of organization, strands which will penetrate the present topics through and through, and will bind the course as a whole into a more excellent unity. Certain large ideas, such as measurement, accuracy and comparison, variation, function and dependence, will render the course which is threaded upon them a more worthy contribution to successful living.

In summary, then, I see the arithmetic curriculum changing under the impact of a new psychology from analysis and drill to synthesis and the doctrine of meaning. I observe that a developmental program is being sought, as before, but this time by the offering of whole thoughts and a spiralized treatment of value to the individual—not by a flight from arithmetic—an avoidance mechanism—not by a postponement of the subject as a whole and as to its parts. I deprecate this flight and postponement as applied to the newer objectives and procedures. I grant that the flight has been more or less justified by the meager results hitherto secured but I foresee better results and restored confidence. I suggest that advances are especially needed in respect to the middle grades and finally I believe that a more powerful and effective course will be available when certain large mathematical ideas become strands reaching through the course from beginning to end.

◆ THE ART OF TEACHING ◆

An Introduction to Determinants in Second Year Algebra

By Norman E. Dodson
Cocoa, Florida

Pedagogically it may be profitable at times to sacrifice historical and independent presentation of topics in elementary mathematics. If better understanding and readier acceptance of a new topic can be secured by its presentation not as a separate and new topic, but as something discovered to help in handling some present problem or topic, it may be wise thus to present the new topic.

The subject of determinants as first presented to students is not presented fully, and it becomes necessary later in a career in mathematics to be presented with another and fuller notion of determinants—fourth and higher order determinants. In view of the actual present situation it may not be a too unpardonable sin to use the following presentation of determinants in second year algebra.

The student should be given an insight into one of the values of determinants as a possible source of the derivation of the subject. This statement applies equally as well to the quadratic formula and to other topics in elementary mathematics.

I have found that the following presentation is effective in aiding memory, understanding, appreciation, and in fostering a type of inquisitive attitude not of an adverse nature.

We have found that in solving a set of simultaneous equations such as

$$2x+3y=7$$
$$3x+4y=10$$

it has been convenient to multiply each term of each of the equations by the coefficient of y in the other equation, thus making it possible to eliminate y and solve for x. We may likewise eliminate x and solve for y.

We found in solving literal simultaneous equations that it was almost necessary that we multiply by the coefficients of x or of y so as to eliminate x or y and solve for y or x. We have found that by use of letters as numbers we can work out formulas. Let's solve completely this set of literal simultaneous equations,

$$ax+by=c$$
$$dx+ey=f.$$

(The teacher works this set separately for x and for y at the board.)

If these answers for x and for y are written down under the set of equations

$$ax+by=c$$
$$dx+ey=f$$

$$x=\frac{ce-fb}{ae-db}; \qquad y=\frac{af-dc}{ae-db}$$

we may be able to see something that will make it possible to remember with more ease these formulas for x and for y without using this long process of elimination of x or y.

The first thing you notice is that the denominator for x is the same as that for y. You may have noticed in your work with purely literal equations that the numbers which appear in the denominators for x and for y are the coefficients of x and y. (This and all else of a detail nature should be clearly pointed out at the board.) Perhaps you have noticed also that the numerator of x does not contain the coefficients of x and that it has instead the terms on the right of the equations

which are not coefficients. This is also true in the case of y. (See below for carefulness in this part of the presentation.)

Let's write down the coefficients of x and y that belong in the denominator of each in the same arrangement that they have in the equations, and enclose them in lines

$$\begin{vmatrix} a & b \\ d & e \end{vmatrix}$$

as a symbol of what we want. We know already that we want $ae-db$. It is most convenient then to define our symbol to mean $ae-db$. (This determinant should be written down by writing the letters a, e, d, b in this order so as to aid the student in memory and appreciation.) This is easy to remember and easy to use, since it is the first number times the last—ae—minus the product of the other two—db—which make the other diagonal of the arrangement.

Let's see if we can write the numerator of x in some similar way. (The teacher may allow a bright student to do this.) We want $ce-fb$. By the definition of

$$\begin{vmatrix} a & b \\ d & e \end{vmatrix}$$

we should write

$$\begin{vmatrix} c & b \\ f & e \end{vmatrix}$$

(The teacher should write c, e, f, b in this order.) We see clearly that this is the same as the denominator just written except that c and f have replaced a and d, the coefficients of x. Now the formula or equation for x becomes

$$x=\frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}.$$

Now let's rewrite the numerator of y in this new symbol of ours. It is

$$\begin{vmatrix} a & c \\ d & f \end{vmatrix},$$

and the equation for y becomes

$$y=\frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}.$$

You see easily that the numerator for y is the same as its denominator except that the constants c and f have replaced the coefficients of y.

Let's try this formula on a numerical problem we have already worked to see if and how it simplifies our work. (This should be done at the board under the teacher's leadership.) This helps us to remember how to work some simultaneous equations, and it helps in understanding other topics in algebra.

The teacher should study the class closely during this presentation so as to avoid premature and undue emphasis. He should wait on the understanding and appreciation of his class and not hurry too much. He should allow and encourage the students to precede him in the thinking and discovery of some of the details of the topic.

Preliminary Report of the Joint Commission!

The Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics on "The Place of Mathematics in Secondary Education" has issued a Preliminary Report in two parts. The report is being issued in a preliminary form in order to secure the comments and suggestions of teachers, administrators, and educators. Persons desiring to receive copies can do so by sending ten cents (coins preferred) for each part to the Chairman, Professor K. P. Williams, Indiana University, Bloomington, Indiana.

National Council of Teachers of Mathematics

Fifth December Meeting Held Jointly with The Mathematical Association of America The American Association for the Advancement of Science Williamsburg, Virginia, December 29 and 30, 1938

Thursday, December 29—6:45 P. M.

Dinner Meeting with The Mathematical Association and The Mathematical Society, College Dining Hall.

Arithmetic Section, R. L. Morton, Presiding

Friday, December 30—9:30 A. M., Washington Hall, Room 100

T. G. Foran: An Experimental Study of the Relation of Home Work to Achievement in Arithmetic.

H. E. Benz: Some Preliminary Considerations Relating to Arithmetic in the High School.

B. R. Buckingham: The Contributions of Arithmetic to a Liberal Education.

Secondary School Mathematics Section, H. C. Christofferson, Presiding

Friday, December 30—9:30 A. M., Washington Hall, Room 200

1. Herbert ReBarker: Meaningful Mathematics.
2. Francis G. Lankford Jr.: An Analytical Study of High School Plane Geometry.
3. M. L. Hartung: The Evaluation of Achievement in High School Mathematics.
4. K. P. Williams: The Report of the Joint Commission on The Place of Mathematics in Modern Education.

Teacher Training Section, A. J. Kempner Presiding

Joint Meeting with The Mathematical Association of America

Friday, December 30—2:00 P. M., Washington Hall, Room 200

1. A. A. Bennett: A College Mathematics Teacher Views Teacher Training.
2. F. L. Wren: The Professional Preparation of Mathematics Teachers.
3. R. L. Morton: Mathematics in the Training of Arithmetic Teachers.

Reservations in the College Dormitories at a very nominal price. Send requests to Dr. John M. Stetson, 232 Jamestown Road, Williamsburg, Virginia, or to the Committee on Local Arrangement and Exhibits:

1. Francis G. Lankford, Jr., University of Virginia, Chairman
2. Dr. T. McN. Simpson, Jr., Randolph Macon College, Ashland.
3. Miss Mary Scott Howison, William and Mary, Williamsburg.
4. Miss Carrie Taliaferro, State Teachers College, Farmville.

Tentative Program of the Twentieth Annual Meeting of the National Council of Teachers of Mathematics Cleveland, Ohio, February 24–25, 1939

THEME: FUNCTIONING MATHEMATICS

Friday—9:00 A. M.

Board of Directors Meeting.

Friday—2:00 P. M.*

(1) Teacher Training, J. P. Everett presiding.

(2) Mathematics and Its Uses, H. C. Christofferson presiding.

Friday—4:00 P. M.

(3) Tea, Get-acquainted hour, and See-exhibits hour.

Friday—7:30 P. M.

(4) General Meeting.

Speakers—Superintendent Lake, Commissioner Studebaker, W. W. Beatty, and M. L. Hartung.

General Theme: Modern Phases of Mathematics Teaching.

Saturday—8:30 A. M.*

(5) Arithmetic in 1939—Panel, J. T. Johnson presiding.

(6) Secondary School Progressive Mathematics, M. L. Hartung presiding.

10:15 A. M.*

(7) Junior High School—Cincinnati Mathematics Club in charge.

(8) Senior High School—Cleveland

* Active participation by listeners is invited. This means that questions and discussion from the floor is urged in each sectional meeting.

Mathematic Club in charge, H. E. Grime presiding.

12:00 M.

(9) Luncheon for Delegates and Representatives—Topic: The Status of Mathematics. Mrs. Miller, presiding.

2:00 P. M.†

(10) New Developments in Secondary School Mathematics—Panel—R. Schorling, presiding; W. A. Brownell, presiding;

(11) Arithmetic, W. A. Brownell, presiding.

4:00 P. M.

(12) Business Meeting.

6:00 P. M.

(13) Discussion Dinner, Directors, Speakers, and Honored Guests presiding.

8:00 P. M.

(14) Entertainment, Cleveland in charge.

Headquarters—Carter Hotel.
Exhibit Chairman—H. E. Grime.
Local Chairman—A. Brown Miller.
Reservations Chairman—J. J. Rush.

† An additional section may be added.

The National Council of Teachers of Mathematics

Official Notice

As Secretary of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers, I officially announce the annual election of certain officers of the National Council, said election to take place at Cleveland, Ohio, on Saturday, February 25, 1939. Article III Section 7 of the by-laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity, through announcement in the official journal, to suggest by mail for the guidance of the directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the secretary of the board of directors shall send to each member an official ballot giving the names of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the board of which the secretary shall be chairman. The election shall be by mail or in person and shall close on the date of the annual meeting."

At the Atlantic City meeting, February 26, 1938, of the National Council the nominating committee consisting of the two most recent ex-presidents and the secretary as chairman (for this year J. O. Hassler, Martha Hildebrandt, and Edwin W. Schreiber), was instructed to prepare an official ballot suggesting two elegible candidates for each elective office and reserving a blank space for a third prospective candidate which may be written in by the voter. The officers to be elected at the Cleveland meeting are: Second Vice President, 1939–40, and three Directors, 1939–41. Since there is to be no primary ballot, affiliated organizations, local clubs, or individual members who wish to present names to appear on the official ballot should do so by writing to the Secretary immediately. The official ballot will be sent to members through the mail the first week in January.

The periods of service of the officers of the National Council, from its organization in 1920 to the present time, are printed below.

Edwin W. Schreiber, *Secretary*

The National Council of Teachers of Mathematics

Organized 1920—Incorporated 1928

Periods of Service of the Officers of the National Council

Honorary Presidents

*H. E. Slaught, Chicago, Ill., 1936–1937

Presidents

C. M. Austin, Oak Park, Ill., 1920
J. H. Minnick, Philadelphia, Pa., 1921–1923
Raleigh Schorling, Ann Arbor, Mich., 1924–1925
Marie Gugle, Columbus, Ohio, 1926–1927
Harry C. Barber, Exeter, N. H., 1928–1929
John P. Everett, Kalamazoo, Mich., 1930–1931
William Betz, Rochester, N. Y., 1932–1933
J. O. Hassler, Norman, Okla., 1934–1935
Martha Hildebrandt, Maywood, Ill., 1936–1937
H. C. Christofferson, Oxford, Ohio, 1938–1939

Vice-Presidents

H. O. Rugg, New York, City, 1920
E. H. Taylor, Charleston, Ill., 1921
Eula Weeks, St. Louis, Mo., 1922
Mabel Sykes, Chicago, Ill., 1923
Florence Bixby, Milwaukee, Wis., 1924
Winnie Daley, New Orleans, La., 1925
W. W. Hart, Madison, Wis., 1926
C. M. Austin, Oak Park, Ill., 1927–1928
Mary S. Sabin, Denver, Colo., 1928–1929
Hallie S. Poole, Buffalo, N. Y., 1929–1930
W. S. Schlauch, New York City, 1930–1931
Martha Hildebrandt, Maywood, Ill.,1931–1932
May A. Potter, Racine, Wis., 1932–1933
Ralph Beatley, Cambridge, Mass., 1933–1934
Allan R. Congdon, Lincoln, Neb., 1934–1935
Florence Brooks Miller, Shaker Heights, Ohio, 1935–1936
Mary Kelly, Wichita, Kansas, 1936–1937
John T. Johnson, Chicago, Ill., 1937–1938
Ruth Lane, Iowa City, Iowa, 1938–1939

Secretary-Treasurers

J. A Foberg, Chicago, Ill., 1920–1922, 1923–1926, 1927, 1928 (Appointed by Board of Directors)
Edwin W. Schreiber, Ann Arbor, Mich., and Macomb, Ill., 1929– (Appointed by the Board of Directors)

Committee on Official Journal

John R. Clark, Editor, 1921–1928
W. D. Reeve, Editor, 1928–
Vera Sanford, 1929–
H. E. Slaught, 1928–1935
W. S. Schlauch, 1936–

Directors

Marie Gugle, Columbus, Ohio, 1920–1922, 1922, 1928–1930, 1931–1933
Jonathan Rorer, Philadelphia, Pa., 1920–1922
Harry Wheeler, Worcester, Mass., 1920–1921
W. A. Austin, Fresno, Cal., 1920–1921
W. D. Reeve, Minneapolis, Minn., 1920, 1926–1927
W. D. Beck, Iowa City, Iowa, 1920
*Orpha Worden, Detroit, Mich., 1921–1923, 1924–1927
C. M. Austin, Oak Park, Ill., 1921–1923, 1924–1927, 1930–1932
Gertrude Allen, Oakland, Cal., 1922–1924
W. W. Rankin, Durham, N. C. 1922–1924
Eula Weeks, St. Louis, Mo., 1923–1925
W. C. Eells, Walla Walla, Wash., 1923–1925
*Harry English, Washington, D. C., 1925–1927, 1928–1930
Harry C. Barber, Boston, Mass., 1925–1927, 1930–1932, 1933–1935
*Frank C. Touton, Los Angeles, Cal., 1926–1928
Vera Sanford, New York City, 1927–1928
William Betz, Rochester, N. Y., 1927–1929, 1930–1931, 1934–1936, 1937–1939
Walter F. Downey, Boston, Mass., 1928–1929
Edwin W. Schreiber, Ann Arbor, Mich., 1928–1929
Elizabeth Dice, Dallas, Tex., 1928, 1929–1931
J. O. Hassler, Norman, Okla., 1928, 1929–1931, 1933
John R. Clark, New York City, 1929–1931
Mary S. Sabin, Denver, Colo., 1929–1930, 1931–1933
J. A. Foberg, California, Pa., 1929
C. Louis Thiele, Detroit, Mich., 1931–1933
Mary Kelly, Wichita, Kan., 1932
John P. Everett, Kalamazoo, Mich., 1932–1934
Elsie, P. Johnson, Oak Park, Ill., 1932–1934
Raleigh Schorling, Ann Arbor, Mich., 1932–1934
W. S. Schlauch, New York City, 1933–1935
H. C. Christofferson, Oxford, Ohio, 1934–1936, 1937–1939
Edith Woolsey, Minneapolis, Minn., 1934–1936, 1937–1939
Martha Hildebrandt, Maywood, Ill., 1934–1935
M. L. Hartung, Madison, Wis., 1935–1937, 1938–1940
Mary A. Potter, Racine, Wis., 1935–1937
Rolland, R. Smith, Springfield, Mass., 1935–1937, 1938–1940
E. R. Breslich, Chicago, Ill., 1936–1938
L. D. Haertter, Clayton, Mo., 1936–1938
Virgil S. Mallory, Montclair, N. J., 1936–1938
Kate Bell, Spokane, Wash., 1938–1940

* Deceased

EDITORIALS

Improvement of Instruction

In an age when the high school population has changed enormously in character from what it was thirty years ago, we need not only a wise adaptation of our teaching material to the needs and abilities of high school pupils, but we need to emphasize as never before the need for improving instruction in the schools. With the influx of pupils into the high school in recent years, we have been compelled to permit people to teach mathematics who really were not properly equipped to do so. As a result, we have an oversupply of teachers. But we do not have an oversupply of good teachers.

Nobody can honestly say that we have not made progress in developing better teachers for the schools, but if the next generation of pupils is to get a square deal or even a *new deal*, we must place a much stronger emphasis upon the quality of teachers in the schools. How this is to be brought about in *mathematics* is a question that should at once be considered by the National Council of Teachers of Mathematics.

The enrollment in high schools is approaching a plateau, and now seems to be a good time to begin thinking how we are going to adjust our teaching to the enormous individual differences in native ability, experiences, and interests of high school pupils.

Report of the Joint Commission

The May issue of The Mathematics Teacher carried an announcement of the publication of Parts I and II of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics on "The Place of Mathematics in Secondary Education" (see page 337 of this issue). Up to date our readers have been slow to write to the chairman, Professor K. P. Williams, of Indiana University, for copies of this tentative report. If the work of the Commission is to have a maximum amount of influence, teachers of mathematics should read this tentative report and make constructive criticisms to Professor Williams upon it.

If teachers of mathematics themselves do not generally keep in touch with the attempts to reorganize the content of our present mathematics program, we can expect the enrollment in mathematics to continue to decline.

An Appeal to Our Subscribers

With the school year's first salary checks out, heavy mailings of renewed subscriptions have been coming into the office of The Mathematics Teacher. The editor's first joy is in this tangible evidence that the cream of mathematics teachers throughout the country value our magazine. Afterward, but just as important, comes the satisfaction of knowing what funds he has to work with during the coming year in his task of maintaining the Teacher on the same high professional and editorial level that has made it so important in the work of teachers all over the United States and in foreign countries.

The Council would like to have its publication in the hands of every mathe-

matics teacher in the country whether he has paid his subscription or not. However, our rates are so low and our margins of financial safety so slender, that we are no longer able to provide the magazine on any other than a pay-in-advance basis. Exceptional allowance has been made for members whose subscriptions expire in May. Those who failed to renew in May were carried on our books over the summer and received the October number. However, in other months there is no prospect of a long summer without salary just ahead to inhibit payment; so we hope renewals will be accompanied by check or money order.

Moreover, we can render better service if members will return the notification card with their remittance. The reason is that the cards are addressed exactly as subscribers are listed in our file. If our record has "John Smith, Head of the Mathematics Department, Cloverdale High School," and the renewal says simply "Head of the Mathematics Department, Cloverdale High School," we may not connect the renewal with John Smith. Then he will be chagrined to receive his magazine as Head of the Mathematics Department in the same mail with a card telling John Smith that he is to be dropped from the mailing list for non-payment of dues. It may seem as though common sense on our part would prevent that absurdity, but, with a mailing list of thousands, our clerical procedures are necessarily a mechanical process in which memory and common sense can play little part.

Finally, the sooner we have changes in addresses, the fewer the chances are of the magazine being returned to us unclaimed.

Pythagoras

Pythagoras, thou ancient honored man,
Philosopher, thou Sage of Samos isle,
Geometer, measurer of many a span,
Journeyer to far Egypt and the Nile,
Believer in the system of numbers,
Savant, suscriber to a fancy-flight—
The thought that man's soul in a beast slumbers—
Dispeller of darkness, Bringer of light,
Thou with thy monochord, thy stretchèd wire,
Thy intervals, octave and dominant,
Thou wert a player on the eight-string'd lyre.
'Twas thou alone, of all of us, whose ears,
Attuned, discerned the music of the spheres.

By Felicite Mueller
Sheboygan, Wisconsin, High School

IN OTHER PERIODICALS

By Nathan Lazar
Alexander Hamilton High School, Brooklyn, New York

1. Bell, E. T., "Father and Son: Wolfgang and Johann Bolyai." *Scripta Mathematica*. (a) 5: 37–44. January, 1938. (b) 5: 95–100. April, 1938.

A fascinating memoir on one of the inventors of non-Euclidean geometry—Johann Bolyai. The author also describes with great insight and sympathy the rôles played by Wolfgang Bolyai and Gauss in that epoch-making intellectual event.

2. Black, M., "The Relevance of Mathematical Philosophy to the Teaching of Mathematics." *The Mathematical Gazette*, 22: 149–163. May, 1938.

In the limited space at his disposal the author gives a clear summary of many of the problems and currents of thought in mathematical philosophy and symbolic logic. The contributions of Peano, Russel, and Whitehead to the reduction of pure mathematics to arithmetic are indicated with the consequent unification of the whole of mathematics into one gigantic system. Many of the subsequent problems, such as the paradoxes and antinomies, are given in some detail, and the various plans for their resolution are analyzed.

Unfortunately only one-half page is devoted to that topic which would interest teachers of mathematics, "The Relevance of Mathematical Philosophy *to the Teaching of Mathematics*."

3. Brand, Louis, "The Significance of Mathematics in the Physical Sciences." *School Science and Mathematics*, 38: 607–613. June, 1938.

This paper contains a brief survey of the various geometries that have arisen in the history of mathematics and of their usefulness in the physical sciences. A similar study is made of various algebras—of complex numbers, of quaternions, of tensors, and of matrices—and their importance for current developments in physics is pointed out.

4. Bubb, Frank W., "On the Study of Mathematics." *School Science and Mathematics*, 38: 639–644. June, 1938.

A radio address delivered over KSD, St. Louis, on February 2, 1938, and intended primarily for parents without any mathematical background.

5. Ginsburg, Jekuthiel, "Gauss's Arithmetization of the Problem of Queens." *Scripta Mathematica*, 5: 63–66. January, 1938.

A problem well known in chess is to put eight queens on a chessboard so that no one can take the other. An equally well known problem in recreational mathematics which goes back to Gauss is to find such an arrangement of the numbers from 1 to 8, that different sums are obtained: (1) when these numbers are added to the numbers 1, 2, · · · 8 in their natural order, (2) when they are added in order to the numbers, 8, 7, · · · 1.

After enumerating the possible solutions to the above problems, the author points out the perfect isomorphism between the *chess* problem and the corresponding *arithmetical* problem. He then proceeds to show that the relation between the two problems may be shown by purely *geometrical* considerations.

6. Hogben, Lancelot, "Clarity Is Not Enough." *The Mathematical Gazette*, 22: 105–122. May, 1938.

An address by the well known author of *Mathematics for the Million* on the "Needs and Difficulties of the Average Pupil."

The following are some of the high spots of the address: " . . . the teacher's job is less to make things clear than to give his pupils a powerful incentive for getting things clear for themselves. Three obstacles which he has to surmount are easy to recognize. The first is the paralyzing sense of *unfamiliarity*, which I have called the 'So what?' reaction. The second is a sense of *intellectual inferiority*, which discourages further effort. The third is a disinclination of studies with no explicit *practical outcome*."

Under the heading "Some Overdue Reforms," he makes the following remarks: "I believe that much could be done to improve the teaching of mathematics without any radical change in the examination system or in the general policy of education . . . . To meet the new demands for mathematical proficiency, I venture to suggest that the most urgent syllabus reform is a wholesale reduction of formal plane geometry to make way for a much earlier introduction to trigonometry, analytic geometry, calculus, and solid figures. I also suggest that this might be helpfully supplemented by closer

co-operation with the teaching of geography and elementary descriptive astronomy in the schools."

7. Pyle, John O., "Some Other Ideas about the Subtraction of Signed Numbers." *School Science and Mathematics*, 38: 676–679. June, 1938.

Mr. Pyle takes issue with a remark made by Mr. Tate in an earlier issue of the same magazine that "Neither arithmetic nor out-of-school experience provides a background for algebraic subtraction," and proceeds to show the close connection between algebra and arithmetic, and the adequacy of out-of-school experience for illustrative material.

"The addition problem is: given two addends, if the number indicated by one is counted on to the other, with what number do we stop? This last number is called the sum of the two given numbers. Either addend can be used for the starting number; it makes no difference in sum.

"The subtraction problem is: given a subtrahend and a minuend, what number must be counted on to the subtrahend to reach the minuend? The answer in this case is called the remainder. If the minuend be taken for the starting number, the remainder is not the same, unless the subtrahend and minuend are equal numbers. In that case the remainder is always zero."

The interpretation of the process of algebraic subtraction is given in terms of thermometer readings. "The use of signed numbers in problems requiring subtraction to compute changes in other phenomena is as simple as that given here for changes of temperatures."

8. Reeve, William D., "The Place of Mathematics in Modern Education." *Scripta Mathematica*, (a) 5: 23–31. January, 1938. (b) 5: 111–116. April, 1938.

The traditional reasons for teaching mathematics are enumerated and discussed in some detail: (a) its usefulness, (b) its cultural value, (c) its disciplinary value. The real reasons for the prestige of mathematics are then given: (a) the wonder motive, (b) the force of tradition, (c) the possibility of teaching logic through geometry, (d) mathematics as a fine art, (e) the close relation between mathematics and science. But in order to show why mathematics should have an important place in the school, it would be necessary to state a list of objectives. After pointing out some of the difficulties that beset any attempt to determine the objectives of any subject, the author proceeds (in the April issue) to a careful and specific enumeration of the work in mathematics to be done in each of the following: seventh, eighth, and ninth grades, and senior high school.

Both articles contain many apposite quotations and bibliographical references.

9. "The Relative Value of Pure and Applied Mathematics." *The Mathematical Gazette*, 22: 132–147. May, 1938.

A report of a discussion that took place at the Annual Meeting of the [British] Mathematical Association in January, 1938. Ten speakers from various universities, colleges, and secondary schools participated.

10. Sanders, S. T., "Mathematics—Human Laboratory Instrument." *National Mathematics Magazine*, 12: 368–370. May, 1938.

"If by magic or other means there could be created at the end of the present school year a nation-wide situation in which would be found recorded in no school or college a single failure in mathematics, the present tide of popular opposition to mathematics would recede with startling swiftness. In lieu of the present annual outpouring from our schools of thousands of young people whose hostility is born of repeated and humiliating failure in it, would be multitudes voicing their praise and admiration for it."

After pointing out certain desirable changes in the objectives of mathematics and in the teaching procedures, the writer concludes with the following hypothesis: "Mathematics, being in its last analysis a type of rigorous but normal thinking, should be subject to mastery as a method of thinking by every mind not *abnormal* or *subnormal* in character, i.e., by *any normally constituted* intelligence."

11. "Teaching the Complete Duffer." *The Mathematical Gazette*, 22: 164–179. May, 1938.

A report of a discussion held at the Annual Meeting of the [British] Mathematical Association in January, 1938, on a topic familiar to us in the United States in a different form—"Teaching Those with a Low I.Q."

The following are some of the comments made by one of the participants:

"I don't believe the *complete* duffer in mathematics exists; just as I am skeptical of one who is said to have *no* ear for music . . . . There are two motives that we may hopefully appeal to: curiosity and utility . . . . Curiosity first, because in the history of the race that surely came far earlier than utility. Man was asking the why and how of things long before he gained control over them for the purpose of earning his bread and butter . . . . The history of mathematics may be a fruitful source of suggestion . . . . How does

a sundial work? Why are there 360 degrees in a circle (except in Germany)? How did our numerals come to have their present shapes, and how did they improve over other systems of notation in the past?"

Another participant made the following confession: "I found a duffers' class more interesting than a class of average boys. They showed a greater variety of mental and temperamental qualities. And I learnt from them a good deal of what was useful in meeting the difficulties that arise in an average class."

NEWS NOTES

Dr. Saunders MacLane has been appointed Assistant Professor of Mathematics at Harvard University beginning September 1, 1938.

The Women's Mathematics Club of Chicago and Vicinity held a "Poster Luncheon" on May 14 at The Club Women's Bureau, Mandel Brother's Store. One hundred and ninety-three mathematics posters were on display from thirty-seven high schools. The judges were Mr. Daniel Cotton Rich, The Art Institute; Mrs. Ruth Ray, The Ray Schools; Mr. F. C. Brown, Museum of Science and Industry; Mrs. E. Ebert, Cook County Schools; and Paul Olson, Business Men's Art Club. Individual prizes were awarded for the two best rendered subjects, and to the school sending the best group. A short program followed the luncheon. Miss Neva Anderson of Evanston High School talked on "opportunity classes," and students from Evanston presented a skit, "X on the Spot," under the direction of Miss Louise Neal. The following were unanimously elected for the coming year: President, Miss Hubler, Tilden High School, Chicago; Vice-President, Miss C. Haertle, Englewood High School, Chicago; Secretary, Miss Plapp, Kelly High School, Chicago; Treasurer, Miss H. White, Englewood High School, Chicago.

IONIA J. REHM, *Publicity Secretary*

The William Lowell Putnam Prize Scholarship for 1938 has been awarded to Mr. I. Kaplansky of the University of Toronto. This scholarship is awarded annually by the Division of Mathematics at Harvard University for study at that University to one of the first five contestants in the William Lowell Putnam Mathematical Competition. Mr. Kaplansky plans to use this award during the academic year 1939–40.

Messrs. John W. Green of the University of California and Donald T. Perkins of Yale University have been appointed Benjamin Peirce Instructors in Mathematics at Harvard for the academic year, 1938–39.

The Women's Mathematics Club of Chicago and vicinity held a luncheon meeting Saturday, March 26th, at Mandel's Ivory Room. Miss Martha Hildebrand reported on the National Council meetings at Atlantic City. Dr. H. W. Bailey, University of Illinois, was the guest speaker and had for his topic, "Mathematics in Illinois High Schools."

IONIA J. REHM, *Publicity Secretary*

A joint meeting of the Range Mathematics and Science Clubs was held in the Senior High School of Tower, Minnesota, on Thursday, April 28th. The program committee consisting of Miss Angela Mayerle, Mr. Ralph Iverson, and Mr. J. S. Brula, all of the Tower Senior High School, arranged the following program: Address, Dr. George Selke, President, St. Cloud Teachers College; Address, Dr. C. O. Bemis, Professor of Mathematics, St. Cloud State Teachers College; Illustrated lecture by Miss Evelyn Hoke, Chisholm Senior High School, on her experiences at the University of Michigan Biological Station. Mr. K. C. Satterfield, principal of the Tower Senior High School, served as toastmaster. During the business meeting of the mathematics teachers, the following officers were elected for the ensuing school year: Mr. H. G. Tiedeman, Mt. Iron, President; Mr. Ole Schey, Gilbert, Vice-President; Mrs. Etna Gandsey, Virginia, Secretary and Treasurer.

Members of the American Mathematical Society enjoyed a four-day Jubilee Meeting in New York City, September 6th through 9th. Opportunity was provided not only to review the progress which American mathematics has made in the last half-century, but also to visit with guides some of the interesting places in New York and the vicinity.

On the serious side a distinguished list of speakers included President Nicholas Murray Butler, of Columbia University, R. C. Archibald, G. D. Birkhoff, E. T. Bell, G. C. Evans, E. J. McShane, J. F. Ritt, J. L. Syne, T. Y. Thomas, Norbert Wiener, and R. L. Wilder. Papers were presented by 105 of the most important scholars in American mathematics.

An equally impressive program of recreation made it possible for mathematicians from all over the country to visit the 1939 Worlds Fair site, to cruise up the Hudson River to the United States Military Academy at West Point, and to enjoy other excursions, sports, and activities.

The regular spring meeting of the Mathematics Section of the California Teachers Association, Bay Section, was held May 7th at International House in Berkeley. There were

sixty hardy mathematicians at the luncheon (hardy, to have declined four other almost equally interesting meetings at the same time and in the very near vicinity). The speaker was Professor Ira B. Cross, of the University of California, whose topic was "Towards an Understanding." His thesis was that chaos and disorganization always precede a new economic order. The implication seemed clear that teachers as trained leaders could help in furthering the understanding of the progress of events.

Announcement was made by Mr. McCarty and Professor Levy of the organization in the near future of a Northern California Section of the Mathematical Association of America. Mr. McCarty also gave a fine progress report of the Publicity Committee of the Mathematics Section.

A number of those present decided to send an order for copies of the preliminary reports of the Joint Commission of the N.C.T.M. and the M.A.A., and to study them at the seminar meetings in September.

Election of officers was as follows: Advisory Chairman, Miss Adeline Scandrett, Mission High School, San Francisco; West Bay Chairman, Miss Una McBean, Horace Mann Junior High School, San Francisco; West Bay Secretary, Miss Frances Dealtry, Horace Mann Junior High School, San Francisco; East Bay Chairman, Mrs. Beatrice Anderson, Alameda High School, Alameda; East Bay Secretary, Mrs. Lucy Pitman, Alameda High School, Alameda.

EMMA HESSE

The Publicity Committee of the Mathematics Section of the California Teachers Association, Bay Section, has made the following report:

The offer of Professor W. D. Reeve to supply free copies of the yearbook of the National Council of Teachers of Mathematics to the *Sierra Educational News* and the *California Journal of Secondary Education* for review has been accepted, and grateful acknowledgment of the favor has been made.

Attention of all teachers of mathematics is called to "A Course in Applied Mathematics for Teachers of Secondary Mathematics," by Cleen C. Richtmeyer, THE MATHEMATICS TEACHER, pp. 51–62, February, 1938.

The Committee recommends for junior college, senior high school, and public libraries books of the following type:

Introduction to Mathematics, Cooley, Gans, Kline, Wahlert. Houghton Mifflin Co., San Francisco. 634 pp. $3.25.

Numerology, E. T. Bell. The Century Co., New York. 187 pp. $2.00.

A Mathematician Explains, Mayme I. Logsdon. University of Chicago Press. 175 pp. $2.00.

The Handmaiden of the Sciences, E. T. Bell. Williams and Wilkins, Baltimore. 216 pp. $2.00.

The Place of Mathematics in Modern Education, the Eleventh Yearbook of the National Council of Teachers of Mathematics. Bureau of Publications, Teachers College, Columbia University, New York. 257 pp. $1.75.

Numbers and Numerals, Smith and Ginsburg. Bureau of Publications, Teachers College, Columbia University, New York. 52 pp.

Mathematics for the Million, Lancelot Hogben. W. W. Norton and Co., New York. 647 pp. $3.75.

Elements d'Algebre Ornementale, El-Milick. Paris.

An Invitation to Mathematics, Arnold Dresden. Henry Holt and Co., New York. 453 pp. $2.80.

Men of Mathematics, E. T. Bell. Simon and Schuster, New York. 592 pp. $5.00.

The Committee recommends that teachers bring books of this type to the attention of administrators.

Articles having to do with the relation of mathematics to other fields of knowledge should be published in general professional magazines, such as *Sierra Educational News* and the *California Journal of Secondary Education*.

Each teacher of mathematics should consider herself a committee of one to encourage the writing of such articles.

The publicity features of the Committee's activities should be continuous.

This report is the work of A. L. McCarty, Chairman, San Francisco Junior College; Elenore Lazansky, University High, Oakland; Mrs. Edythe Samules, Presidio Junior High, San Francisco; A. H. Smith, Balboa High, San Francisco; and C. A. Templeton, Claremont Junior High, Oakland.

Professor Walter W. Hart addressed section meetings of the Nebraska State Teachers Association in North Platte, October 27th, and in Omaha, October 28th. Before the junior high school group of the Southwest Section held at North Platte, Dr. Hart spoke on "Balancing Objectives and Techniques." His topic for the mathematics group meeting was "Improving Instruction in Mathematics."

At the luncheon and afternoon meeting of the mathematics group of the Southeast Section in Omaha the next day, he spoke informally on "Socialized General Mathematics."

Several Benjamin Peirce Instructorships at Harvard University are open for the academic

year, 1939–40. These instructorships are ordinarily awarded to men who have recently received the doctor of philosophy degree or have had equivalent training. Those interested in applying should write to the Chairman of the Division of Mathematics.

Professor Arnaud Denjoy of the University of Paris is in residence at Harvard University for the first half of the academic year, 1938–39, as Exchange Professor from France.

The second annual meeting of the Nebraska Mathematics Teachers Association was held at Hastings College, Hastings, Nebraska, on Saturday, 9:00 A. M., May 7. The following program was carried out:

Carl Thomas, Chadron State Teachers College, President. Address: Dr. A. R. Congdon, University of Nebraska. Report of Meeting of National Council at Detroit, Miss Eva Phalen, Kearney High School. Business Meeting. Report of Committee on Curriculum and Course of Study, Miss Lena Meyer, Kimball High School.

Luncheon was served in the Grill Room, Clarke Hotel. Afterward the following officers were elected for the year, 1938–39. President, Miss Eva Phalen, Kearney High School, Kearney; Vice-President, Miss Mary Doremus, Fremont High School, Fremont; Secretary, Miss Jennie Walker, Norfolk High School, Norfolk; Treasurer, Miss Ellen Anderson, Lincoln High School, Lincoln; Executive Committeeman, District V. Herbert W. Finke, Holdrege High School, Holdrege; Executive Committeeman, District VI, Miss Lena Meyer, Kimball High School, Kimball.

HERBERT W. FINKE

The Louisiana-Mississippi Sections of the Mathematical Association of America and the National Council of Teachers of Mathematics met jointly on March 11 and 12, at Mississippi State College, State College, Miss. The program follows:

Joint preliminary meeting, 2:00 P.M., March 11. Address of welcome by Dean S. B. Hathorn, Mississippi State College.

Louisiana-Mississippi Section of the Mathematical Association of America, 2:30 P.M., March 11. Dorothy McCoy, Belhaven College, Chairman, presiding; W. V. Parker, Louisiana State University, Secretary. The following addresses were given: "A General Theory of Limits," H. L. Smith, Louisiana State University; "Homogeneous Diophantine Equations," A. A. Aucoin, Louisiana State University; "On Tschirnhausen Transformations," W. E. Cox, Mississippi State College; "Book Values of a Specific Bond Purchase," I. C. Nichols, Louisiana State University; "Involutions on a Complex Line," B. E. Mitchell, Millsaps College; "The Differentiability of an Arc," H. T. Fleddermann, Loyola University; "The N Curvatures of a Twisted Curve in Euclidean N-Space," Gordon Walker, Louisiana State University; "The Determination of the Maya Unit of Measure," G. F. Cramer, Tulane, University; "Functions Analogous to Trigonometric Functions," V. B. Temple, Louisiana College; "Some Notes on Divisibility and Direct Division," T. A. Bickerstaff, University of Mississippi; "A Certain Type of Correspondence," E. P. Coleman, Mississippi State College.

Joint banquet, college cafeteria, 7:30 P. M., March 11. Dorothy McCoy, toastmaster. Welcome by President G. D. Humphrey, Mississippi State College. Address, "Important Curriculum Problems in the Teaching of Mathematics," W. D. Reeve, Teachers' College, Columbia University.

Joint meeting of the Louisiana-Mississippi Sections and the National Council of Teachers of Mathematics, 8:30, A. M., March 12. The following addresses were given: "Mean Figures," Bayles Shanks, Millsaps College; "A New Approach to the Solution of the Cubic and Quartic," Eckford Cohen, student, Starkville High School—introduced by C. D. Smith; "The Human Aspect of Mathematics Teaching," S. T. Sanders, Louisiana State University; "Expedient Compromises Between the Traditional Viewpoint and Current Trends in Mathematical Curricula and Instruction," A. C. Maddox, Louisiana State Normal College; "Report of Committee on High School and College Mathematics," P. K. Smith and H. F. Shroeder, Louisiana Polytechnic Institute; "The Joint Commission on the Place of Mathematics in the Secondary Schools," H. E. Buchanan, Tulane University; "A Progress Report on the Work of the Joint Commission on 'The Place of Mathematics in Secondary Education'," W. D. Reeve, Teachers College, Columbia University.

A general discussion, business meetings and election of officers closed the session.

The Men's Mathematics Club of Chicago celebrated Past Presidents' night at the last dinner and meeting of the year held at the Central Y.M.C.A. on May 20. The Past Presidents were guests of the Club in token of the respect and indebtedness due to the men who have carried the burden of perpetuating the ideals of the Club. Each brought a short message.

Professor Walter W. Hart spoke on "General High School Mathematics," a subject to which the Club has given considerable thought and discussion during the past year. Professor

Hart's views, based on mature judgment and sound educational principles, were a climaxing contribution to these discussions.

The Oklahoma Council of Teachers of Mathematics held its annual meeting in Oklahoma City on February 11.

A report on recent textbook adoptions was given during the business session. Only one reversal of the recommendations made at the Council's last meeting occurred, and that in the plane geometry adoption.

The following officers for the coming year were elected: President, James A. Mathews, Oklahoma City; Vice-President, F. L. Divine, Blackwell; Secretary-Treasurer, Kathleen Begley, Tulsa.

During the past year the membership of the Council has increased from 114 to 133 members.

The program of the meeting follows: Report on the present position of Composite Mathematics, John A. Venable. Address, "Mathematics and Its Challenge to High School Teachers," Ralph D. Dorsett; University of Oklahoma. Panel Discussion, arranged by L. W. Lavengood Tulsa, "The Effects of Progressive Education on the Teaching of Mathematics," Dr. J. O. Hassler, University of Oklahoma, leader. "Present and Desired Changes in the Preparation of Mathematics Teachers by our Teacher Training Institutions," Mr. W. T. Short. "Consumer Mathematics in the Scheme of Progressive Education," Mrs. Anne L. Cowan. "Mathematics in the Core Curriculum of Seventh Grade." Mrs. Grace West. "How Does the Philosophy Underlying the Progressive Education Mathematics Program Fit in with General Education?" Mr. H. E. Wrinkle. "Some Constructive Criticisms of Progressive Education." (Besides your own see Bode's article in the January, 1938, *Progressive Education*, p. 7; also, *One Foot on the Ground*, by Cobb, G. P. Putnam's Sons, 1932.) Miss Kate Barbour. The Tulsa Set-up, Mr. L. W. Lavengood.
